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VERIFICATION OF UNIVERSITY CLASS OF DIFFERENT CRITICAL EXPONENTS FOR COMPETITIVE GROWTH MODEL IN 1+1 DIMENSION

SUDEEP KUMAR DAS
Department of Physics
Durgapur Government College

SOMA MITRA
Department of Physics
University of Gour Banga
Malda, West Bengal 732103, India

DIPTONIL BANERJE
Dr. M.N. Dastur School of Materials Science and Engineering
IIEST, Shibpur, West Bengal 711103, India
Corresponding Author: nilju82@gmail.com (D.B.)

ABSTRACT:

In this work we have reported the evolution of rough surface by different competitive growth model in 1+1 dimension. The competition has mainly been made between Random Deposition-Ballistic Deposition (RD-BD) and Random Deposition with Surface Relaxation-Ballistic Deposition (RDSR-BD) model. The influence of a typical growth mechanism on the critical time and interface width has been studied in detail by varying the fractional values (*p*) for a typical model.

Different parameters like roughness exponent, growth exponent or velocity of the surface growth have been studied and a typical dependence of all these parameters on *p*has been found and reported for the first time. It has been found that unlike the previously reported result the growth cannot be described by any single mechanism even for the pure RD, RDSR or BD model rather there exists two distinct crossover times and thus it may be best described by linear combinations of two different growth models.

KEYWORDS: Scaling , Discrete surface growth model, Ballistic deposition, Roughness

INTRODUCTION:

From last few decades analysis of morphology of surface growth and evolution of interfaces, is one of the topic of attraction for the studies of different physical and chemical phenomena that includes snow falling on a slanted glass window, piling of sand on a smooth surface, propagation of fluid or fire front through any paper sheet, bacterial colony growth or growth of thin film by molecular beam epitaxial method and many others[1-3].In practice there are three different ways by which a surface growth can be taken place. These ways are the following:

a) Surface growth considering deposition of a single kind of particles obeying same growth mechanism [1-3]

b) Surface growth considering deposition of a single kind of particles that undergo a deposition (or evaporation) subjected to different growing mechanisms [4,5].

c) Surface growth considering deposition of two or more different kind of particles [6].

Also depending upon the growth mechanism, there are different discrete models which adequately describe surface growth that includesrandom deposition (RD), ballistic deposition (BD), random deposition with surface relaxation (RDSR)solid on solid model (SOS),body centered solid on solid model (BCSOS) and many others [7-9].Other than RD model, all the models are developed on some simple stochastic growth rule obeying nearest neighbor interaction. These models best describe the surface growth involving one kind of particle.To obtain a more realistic surface for describing different natural phenomena more accurately people are developing models called competitive growth where it is considered that a definite growth is taking place with a specific probabilities say *p* whereas other is taking place with probability (*1-p*) [10, 11].

The values of the different scaling exponents for the various universality classes have been reported for 1+1 or 2+1 dimension with like particles or particles with different shapes and sizes obeying certain growth mechanism or some competitive growth models [12].Braunstein and his co-worker studied the probability dependence of different scaling exponent for their competitive growth model that includes single type of particle [13].Numerical studyof the ballistic model for both the sliding as well as sticky particle in different dimensions has been done [14]. Chame and his co-workers have studied the crossover effects in a discrete deposition model with a particular scaling [15]. Muraca et al. reported the universal behavior of the coefficients of the continuous equation in competitive growth models [16]. However, regarding the deviation of the university class behavior of these different exponents for competitive as well as single discrete growth process have not been reported so much. Only D. Jana et.al. reported the non-universal behavior of scaling exponents corresponding to the height fluctuation in (1+1) dimension for a nonlinear discrete growth model that involves extended particles [17].

Motivated by above mentioned literature study, here two kinds of competitive growth model RD-BD (model 1) and RDSR-BD (model 2) have been simulated in 1+1 dimension considering a single type particles along with pure RD, RDSR and BD model. The corresponding scaling exponents have been calculated and tabulated for different system sizes. Also the porosity as well as the growth velocity of the produced surface has been calculated and dependence of all these parameters on fractional probability has been reported. It has been shown that unlike the previously reported result the growth cannot be described by any single mechanism even for the pure RD, RDSR or BD model rather there exists two distinct crossover times and thus it may be best described by linear combinations of two different growth models.

The paper is organized as follows. In section 2, a brief description of the existing discrete growth models along with some basic definitions are given. Also the basic assumption and condition of the model is depicted here. Section 3, describes the results followed by corresponding discussion and conclusion in section 4 and 5 respectively.

MODELING AND SIMULATION:

The roughness of a growing surface can be characterized in terms of W(L,t) which is defined as:

$$W(L,t) = \sqrt{\frac{1}{L}\sum_{i=1}^{L}(h(i,t) - H(t))^2} \quad (1)$$

Where L is the system size,$h(i,t)$ is the height of the i^{th} site at time t and $H(t)$ is the mean heightof the surface given by:

$$H(t) = \frac{1}{L}\sum_{i=1}^{L} h(i,t) \quad (2)$$

For RD model particles are deposited without any surface correlation, hence interface width continuously increases with time.

For BD and RDSR model particles are deposited with surface correlation, hence interface width increaseswith time initially and saturates after a certain time (t_x) called critical time or cross-over time. Thus the time evolution of interface width has two regions separated by critical time, following certain power laws as follows

$W(t) \propto t^{\beta}$ for $t \ll t_x$

$W(L) \propto L^{\alpha}$ for $t \gg t_x$

The critical time depends on the system size as

$t_x \propto L^z$

Where α is the roughness exponent and β is the growth exponent, z is the dynamic exponent and is given by,$z = \alpha/\beta$[18].

The above three relation can be summarized in single expression runs as

$$w(L,t) \sim L^{\alpha} f(\frac{t}{L^z}) \quad (3)$$

Though RDSR and BD models consider surface correlation, still the values of these exponents are different for them as RDSR model falls into linear universality class whereas the BD model falls into nonlinear universality class.

The mathematical form of basic continuum growth equation may be written as follows –

$$\frac{\partial h(x,t)}{\partial t} = G[h(x,t)] + \eta(x,t) \quad (4a)$$

Where, $G[h(x,t)]$ is the deterministic growth term and $\eta(x,t)$is the noise term.

For linear class $G[h(x,t)] = \nu(\nabla^2 h)$ and we get Edward–Wilkinson (EW) [6]equation as

$$\frac{\partial h(x,t)}{\partial t} = \nu(\nabla^2 h) + \eta(x,t) \quad (4b)$$

For nonlinear class $G[h(x,t)] = \nu(\nabla^2 h) + (\lambda/2)(\nabla h)^2$ and we getKardar–Parisi–Zhang (KPZ) [19] equation as

$$\frac{\partial h(x,t)}{\partial t} = \nu(\nabla^2 h) + \frac{\lambda}{2}(\nabla h)^2 + \eta(x,t) \quad (5)$$

Here ν is called surface tension which causes the surface to relax and λ represents lateral growth coefficient.

Here in this work, we have studied different discrete growth models namely RD, RDSR, BD and different competitive growth models in $1+1$ dimension for deposition of single kind of particles.

In all the three cases at any instant *t*, a certain site *i* having height *h(i)* has been chosen randomly and a small particle is released. In case of RD the particle falls and sticks exactly to the position it was released upon thus increasing its height by one unit, i.e. new height of the i^{th} site would be $h(i, t+1) = h(i, t) + 1$. In the second case i.e. in RDSR model the particle so chosen falls on the i^{th} site but can be relaxed to its nearest neighbor if the height of the neighbor is lesser. In case of BD the randomly chosen particle can stick to the nearest neighbor site where it finds the maximum height.

For the competitive growth likeX-Y model (where X, Y stands for RD, BD, RDSR), some deposited particles follow X mechanism with probability *p* and others follow Y mechanism with probability (*1-p*).

In RDSR-BD model, RD is replaced by RDSR and same deposition parameter were used.

For our study, deposition following competitive growth was made for two different lattice size *L* = 64 and *L* = 128for different values of probability*p*. The initial height was made zero. Thetime of deposition was $10^6$with one particle deposited per unit time. The value of fractional probability *p*, was varied from *0* to *1* in steps of *0.25*. The value of interface width and mean height was recorded after each interval of time.

RESULTS:

Fig.1(a-c) shows the variation of ln(w) with respect to ln(t) for all the three models for different system sizes. It is seen from **Fig.1** that though for RD model interface width increases monotonically with β having values ~ 0.5 as reported previously, in other two models there exist two distinct slopes and thus two crossover times,$t_{x1}$and t_{x2} which has not been reported previously.**Table 1** summarizes the values of different scaling parameters as obtained from **Fig.1**.

Fig.2 (a, b) shows the same interface width versus time variation in log-log plot when the system undergoes competitive growth model for two different system sizes where the competition is made between RD and BD model. The fractional probability p = 1suggests that the growth is completely governed by random deposition and p = 0 corresponds system undergoes pure ballistic deposition.

The same characteristics for RDSR-BD competitive model has been shown in **Fig.3** (a, b). The corresponding height profile for a particular system size L = 64 and for all p values has been shown in**Fig.4.**

Table 2 and **Table 3** summarize the different scaling exponents that can be obtained from **Fig.2** and **Fig.3.** In all the cases it is seen that there exists two different slopes and thus two distinct values of growth exponent β also two different crossover times can be found. For RD-BD model at the initial stage of growth up to 1st crossover time t_{x1}, the β value matches well with that is expected from pure RD model for all values of p (except p = 1). As $t>t_{x1}$,theslope changes and takes the value up to a second crossover time t_{x2} that is closed to the value reported for pure BD model. All the values of t_{x1} andt_{x2}increase monotonically as the growth approaches from BD to RD model however for RDSR-BD model it doesn't show any monotonic variation.**Fig.5** shows the variation of W_{sat} with values of p for both the competitive growth model. It has been seen that the W_{sat} increases with p for RD-BD model whereas it shows reverse order when the system undergoes RDSR-BD model.

The growth velocity (v) and average growth velocity ($\bar{v}$) are given by

$$v = \frac{\partial h(x,t)}{\partial t} \quad \text{and} \quad \bar{v} = \frac{1}{L}\int_{x=0}^{L}\left(\frac{\partial h}{\partial t}\right)dx = \frac{\partial \bar{h}}{\partial t} \qquad (6)$$

The fractional porosity of the media is defined as

$$\sigma = \frac{N_v}{N_v+N_p} \qquad (7)$$

Where, $N_v$is the number of voids and N_p is the number of particles deposited.

Fig.6 (a, b) shows the variation of $\bar{v}$ with p for both the competitive growth models and for both the system size L. **Fig.7** shows the variation of $\sigma$with p for the same growth models and system size. It is seen that $\bar{v}$ decreases as system departs more and more from BD for all the system sizes and for both the growth mechanism. Also porosity shows same variation as expected because except BD in all other model particle either sticks to the initial site or finds site with lowest height.

DISCUSSION:

Based on the above results it can be easily concluded that the growth is not being governed by any single process as shown by Jana and Mandal[17]. Thus the growth in the individual time region can be expressed as

$$w(L,t) \sim t^{\beta_1} \ (t < t_{x1}) \qquad (8a)$$

$$w(L,t) \sim t^{\beta_2} \ (t_{x1} < t_{x2}) \qquad (8b)$$

$$w_{sat}(L,t) \sim L^{\alpha} \ (t > t_{x2}) \qquad (8c)$$

with $z_1 = \alpha/\beta_1$ and $z_2 = \alpha/\beta_2$

The subscript '1' and '2' stands for corresponding critical time region.

Thus the complete growth phenomena should be described by the linear sum of these two terms and thus:

$$W(L,t) = L^{\alpha}\left[f_1\left(\frac{t}{z_1}\right) + f_2\left(\frac{t}{z_2}\right)\right] \quad (9)$$

So it can be concluded that for the kinetic growth model as it does not follow any unique scaling relation it thus losses its universality class. It should be noted that from **Table 2** and **3,** one can see that for RD-BD model the initial growth exponent has the value that is almost same as that for pure RD model but as time evolves β takes the value closed to 0.33 which is same if the system follows pure BD model. Thus here the growth can be taken to be linear sum of the RD model and BD model. It may seems to us that initially the system size is much larger compared to numbers of particles thus effectively newly generated one or two particles find sites randomly and thus effectively RD takes place thus in this time regime β has values closed to that expected from pure RD model. As time evolves surface correlation effects come to play and system begins to follow BD model. However this assumption doesn't hold for the second model where initially growth exponent gives value closed to pure BD model and for higher time it changes to the values expected from the pure RDSR model. Thus no definite conclusions can be drawn from these two results except the fact that none of the growth mechanisms even if it is pure RDSR or BD can be best described by single sets of scaling exponent which is not reported previously.

The particle flux is same for both the process but in RDSR the new particle relaxes at the lowest possible position whereas in case of the BD model it searches for the highest height. Thus in case of later the growth rate is much higher. Also in both the cases the surface is initially smooth thus the necessity of the newly arriving particles to relax to the nearest neighbor is rather lesser for $t \le t_{x1}$. As the time evolves roughness increases the newly arriving particles seek new position in order to maintain the definition of corresponding growth process and that changes the slope of the scaling curves from β_1 to β_2 in the time regime $t_{x1} \le t \le t_{x2}$ followed by the saturation of interface width.

Also from **Table 2** and **3** the variation of α with p for both the competitive models can be seen and it is shown that for both the cases α first increases then decreases as system departs more and more from BD model. Variation of dynamic exponent with p in both the critical time regime for both competitive models also shows an overall decrease in the z values for both the time region.

CONCLUSION:

This work reports a comparative simulation study of time evolution of a rough surface generated by three different mechanisms namely random deposition, that with surface relaxation and ballistic deposition. It has been found that the growth cannot be described by any existing scaling relation uniquely and thus losing its universality. There exists in case of both RDSR and BD three distinct growth regimes separated by two critical times. Three regimes behave differently being described by different scaling relations. Thus the entire growth has been described by the linear sum of the two growth equations. Different values of two growth exponents within two growth regimes have been explained physically. It is shown that for competitive model also the different parameters behaving differently according to the models but the growth cannot be described by a single mathematical expression. Different parameters like growth velocity, porosity that have direct influence on different practical phenomenon have been calculated and variation has been shown with the values of factional probability.

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FIGURE CAPTIONS:

Figure–1: Variation of $\ln(W)$ with $\ln(t)$ for different sizes when deposition process follows(a) RD, (b) RDSR and (c) BD model

Figure–2: Variation of $\ln(W)$ with $\ln(t)$ when deposition process follows RD-BD competitive growth model for different system sizes (a) L = 64, (b) L = 128

Figure–3: Variation of $\ln(W)$ with $\ln(t)$ when deposition process follows RDSR-BD competitive growth model for different system sizes (a) L = 64, (b) L = 128

Figure–4: Interface generated due to deposition of 3000 particles on a particular system size L = 64 for different competitive models for different values of fractional probability p

Figure–5: Variation W_{sat} with p for different competitive growth model for L = 64 and L = 128

Figure-6: Variation of $\bar{v}$ with p for (a) RD-BD and (b) RDSR-BD models

Figure-7: Variation of σ with p for (a) L = 64 and (b) L = 128 when growth follows RD-BD and RDSR-BD models

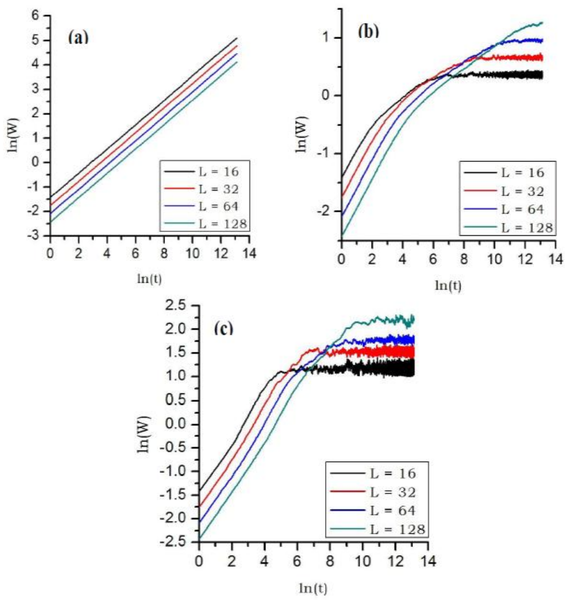


Fig.1: Das et al.

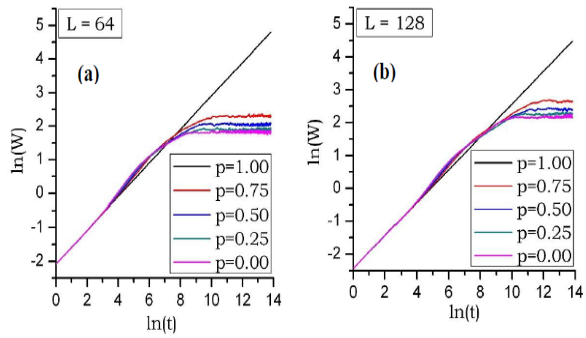


Fig.2: Das et al.

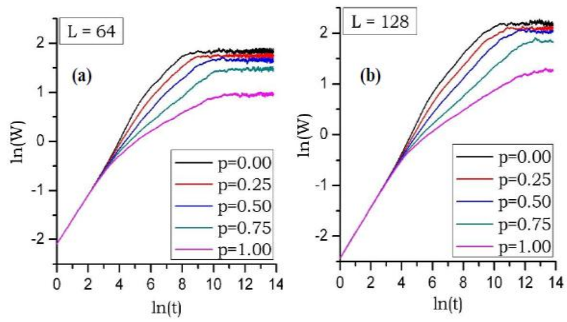
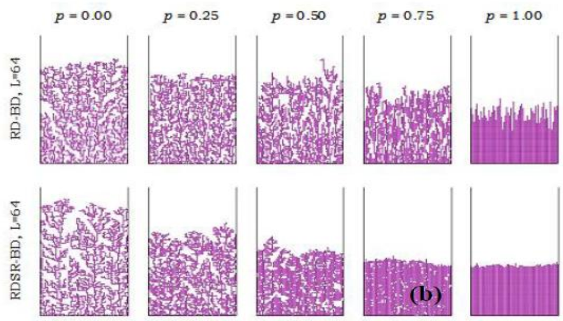


Fig.3: Das et al.



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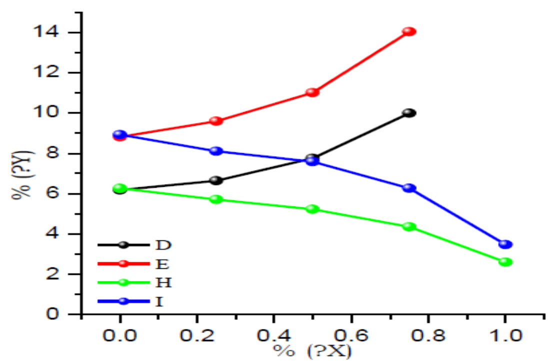

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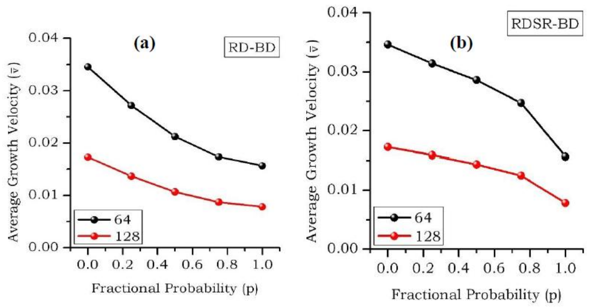

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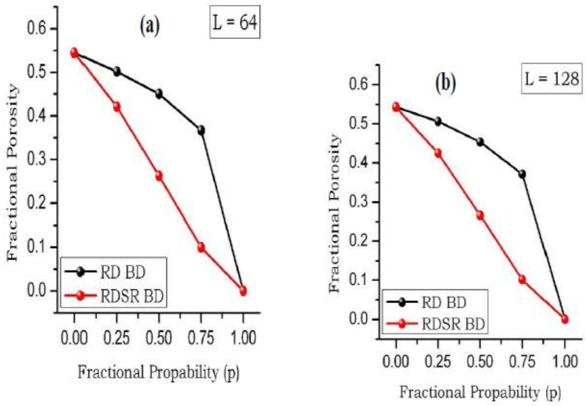

Fig.7: Das et al.

Table 1: The values of different scaling exponents for different system sizes when deposition process follows RD, RDSR and BD model

L	β_1	β_2	W_{sat}	t_{x1}	t_{x2}
Random Deposition (RD)					
16	0.497	---	---	---	---
32	0.497	---	---	---	---
64	0.502	---	---	---	---
128	0.501	---	---	---	---
Random Deposition with Surface Relaxation (RDSR)					
16	0.466	0.245	1.443	6	273
32	0.478	0.247	1.928	16	1241
64	0.476	0.249	2.586	32	7627
128	0.476	0.252	3.579	62	51888
Ballistic Deposition (BD)					
16	0.582	0.337	3.214	59	166
32	0.558	0.335	4.610	158	784
64	0.583	0.324	5.905	217	3748
128	0.536	0.336	8.837	1061	15445

Table – 2: The values of different scaling exponents for different system sizes when deposition process follows RD-BD competitive growth model

L	p	β_1	β_2	W_{sat}	t_{x1}	t_{x2}	α	z_1	z_2
64	0.00	0.561	0.333	6.180	288	3622	0.51	0.91	1.53
	0.25	0.559	0.333	6.643	307	4715	0.53	0.96	1.62
	0.50	0.551	0.333	7.761	431	6832	0.50	0.93	1.52
	0.75	0.538	0.333	9.995	788	12038	0.49	0.91	1.50
	1.00	0.498	-------	-------	-------	-------			
128	0.00	0.555	0.332	8.821	796	17288			
	0.25	0.539	0.322	9.592	867	27032			
	0.50	0.521	0.327	11.009	1121	39729			
	0.75	0.539	0.321	14.028	1655	71851			
	1.00	0.500	-------	-------	-------	-------			

Table – 3: The values of different scaling exponents for different system sizes when deposition process follows RDSR-BD competitive growth model

L	p	β_1	β_2	W_{sat}	t_{x1}	t_{x2}	α	z_1	z_2
64	0.00	0.342	0.251	6.266	1505	4863	0.51	1.47	2.04
	0.25	0.335	0.249	5.712	1739	7528	0.51	1.50	2.08
	0.50	0.330	0.253	5.233	1913	13236	0.54	1.61	2.15
	0.75	0.322	0.254	4.358	281	26423	0.53	1.62	2.06
	1.00	0.328	0.249	2.596	80	7371	0.42	1.27	1.71
128	0.00	0.352	0.249	8.929	5389	26794			
	0.25	0.346	0.241	8.097	9044	38135			
	0.50	0.340	0.248	7.587	13572	68045			
	0.75	0.334	0.261	6.271	794	161593			
	1.00	0.333	0.241	3.467	146	54000			