NOVATEUR PUBLICATIONS
JournalNX- A Multidisciplinary Peer Reviewed Journal
ISSN No: 2581 - 4230
VOLUME 7, ISSUE 5, May. -2021

MODELING THE INFLUENCE OF PATHOLOGIES ON BLOOD FLOW BY MODIFYING THE ELASTIC MODEL FORVASCULAR WALLS

Dilafruz Shukrullaevna Nurjabova
Tashkent University of Information Technologies Karshi Branch,
Department of “Software Engineering”
dilyaranur1986@gmail.com

ANNOTATION:

This article is considered as a review part for creating a mathematical model and software for the pathology of the circulatory system in the cardiovascular system. At the beginning, the cardiovascular system, the organ system, the pathology of the cardiovascular system, the international classifier of diseases of heart disease and blood vessels are studied, and further on what scientists studied blood circulation in the cardiovascular system and what problems were studied.

Keywords: mathematical model and software pathology, organ system, international classifier of diseases of heart disease and blood vessels.

INTRODUCTION:

There is a large number of studies of mathematical models of the heart [Lishchuk VA, 1981; Zavalishin N. N. et al., 1986; Mostkova E.V., 1986; Desai M. D., Saxe na S. C., 1984; Mandel, 1984; Welkowitz W. 1984; Avanzolini G. et al., 1985; Coleman T. G., 1985; Linkens A. D., 1985; Larnard D. J., 1986; Peterson N. et al., 1986; Ohayon J., Oddou C „1987] *, including the author of this work [Lishchuk VA et al., 1965, 1980; Lishchuk VA, 1967-1978], therefore, the description of cardiac activity can be performed only to the extent necessary to include the model of the heart in the general description of the cardiovascular system.

The circulatory system is closed and consists of the heart, arteries, veins and capillaries. We believe that cardiac activity is reproduced by a dynamic model of a four-chambered heart. Each chamber is associated with a spherical reservoir with elastic walls, then a system is composed of the equation of blood flow in the chambers of the heart, the laws of conservation of mass and Poiseuille. A detailed description of the model can be found, for example, in [1]. Networks of arteries and veins of the large and pulmonary circulation are presented in the form of four columns. Each of them docks with one of the chambers of the heart. Vessels are considered to be elastic tubes, the ratio of diameter to length of which is rather small. Let us denote by S section of the vessel; u- cross-section averaged speed; p - transmural pressure; t - time; x- coordinate along the vessel ρ - blood density; ϕ,ψ - specified functions.

METHODS:

Blood is considered a viscous incompressible fluid and its flow in each one-dimensional region can be described by the laws of conservation of mass and momentum [2]:

$$\partial \frac{\partial S}{\partial t} + \frac{\partial (Su)}{\partial x} = \varphi(t, x, S, \bar{u}),$$

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial (\bar{u}^2/2 + \bar{p}/p)}{\partial x} = \psi(t, x, S, \overline{u}),$$

(1.1)

x ∈ [0,l], t ∈ [0,T], where l- vessel length, T is the calculation time. The function ϕ can be used to simulate the inflow / outflow of blood (wall injuries, blood loss), and with the function ψ, the effects of external forces (friction, gravity, etc.). In this work, we put ϕ = 0, and ψ will set the viscous friction and be determined by the formula [3]:

$$\psi = - \quad 16\mu u\eta(\breve{S})/(\breve{S}d^2), \qquad (1.2)$$

$$\breve{S} = S\breve{S}^{-1}, \qquad \eta(\breve{S}) = \begin{cases} 2, & \breve{S}>1, \\ \breve{S}+\breve{S}^{-1} & \breve{S}\le 1, \end{cases}$$

Where $\breve{S}$ is the cross-sectional area of the vessel at p = 0; d is the tube diameter; μ is the coefficient of blood viscosity.

System (1.1) is closed by the equation of state characterizing the elastic properties of the vessel walls:

$$\bar{p} = \rho c_0^2 f(S), \qquad (1.3)$$

$$f(S) = \begin{cases} \exp(S\breve{S}^{-1}-1)-1, & S>\breve{S} \\ \ln(S\breve{S}^{-1}), & S\le\breve{S} \end{cases} \qquad (1.4)$$

c0- - the speed of propagation of small disturbances. The function f (S) is chosen in this way, according to [4]. Generally speaking, it can be specified in various ways [5,6.7], while its graph should be a monotonic S-shaped curve.

The system of equations (1.1) is of hyperbolic type; therefore, at the boundary points of each vessel, the characteristic curves outgoing from the region of integration impose conditions on the solution. These conditions are also called compatibility equations, and to derive them, we represent system (1.1) in divergent form:

$$\frac{\partial V}{\partial t} + \frac{\partial F(V)}{\partial x} = g. \qquad (1.5)$$

Where V = {S, u}, F = {Su, u^2 / 2 + p / ρ }, g = {ϕ, ψ}. Ω^i (i = 1,2) left eigenvectors of matrix (1.1) $A = \frac{\partial F}{\partial V}$ then the characteristic form of system (1.1) is as follows:

$$\omega_i\left(\frac{dV}{dt}\right)_i = \omega_i\left(\frac{\partial V}{\partial t} + \lambda_i\frac{\partial V}{\partial x}\right) = \omega_i g_i i = 1{,}2, \qquad (1.6)$$

λ_i — the eigenvalues of the matrix are the total derivative along the i-th characteristic curve. The eigenvalues are calculated from the equation det (A - λE) = 0, where E is the identity matrix, and are equal to

$$\lambda_i = u + (-1^i)\sqrt{\frac{S\partial p}{\rho\partial S}} = u + (1)^i c_0\sqrt{S\frac{\partial f(S)}{\partial S}}, i = 1.2.$$

Explicit expression for own values:

$$\lambda_i = u + (-1)^i c_0\sqrt{(S)}\begin{cases} \sqrt{\frac{1}{\hat{S}}\exp\left(\frac{S}{\hat{S}}-1\right)}, S>\hat{S} \\ \sqrt{\frac{1}{S}}, S\le\hat{S} \end{cases} = u + (-1)\begin{cases} c_0\sqrt{\frac{S}{\hat{S}}\exp\left(\frac{S}{\hat{S}}-1\right)}, S>\hat{S} \\ c_0, S\le\hat{S} \end{cases}$$

i = 1,2 In addition, from the condition ωi(A_k – λiE) = 0, which the left eigenvectors ωi satisfy, one can find their analytic form:

$$\omega_i = \left\{\sqrt{\frac{1\partial p}{\rho\partial S}}, (-1)^i\sqrt{S} = \begin{cases} c_0\sqrt{(1/\hat{S})\exp(S/\hat{S}-1)}, & S>\hat{S} \\ c_0, (-1)^i S, & S\le\hat{S} \end{cases}\right., i = 1{,}2. \qquad (1.8)$$

$c = \sqrt{\frac{S\partial p}{\rho\partial S}}$ - the local velocity of propagation of elastic waves in the medium (the velocity of sound). The work considers only subsonic currents (u <c), characteristic of blood circulation in normal conditions and in most pathologies. It can be seen from formula (1.7) that each point on the edge, including the boundary point, leaves two characteristics. In the cases under consideration, one of them has a positive inclination to the O_x axis, the other is negative. The characteristics emerging from the region of integration at the ends of the vessel specify conditions (1.6) in them, with i = 1 at the inlet to the vessel and i = 2 at the outlet. Thus, the solution of system (1.1) at each boundary point must satisfy the compatibility equation (1.6) and some other additional condition. When analyzing hyperbolic systems of equations, characteristic variables are often used, they are also Riemann invariants. These parameters W_1 and W_2 are constant along the characteristic curves and have the following expression:

$$\frac{\partial W_i}{\partial V} = \omega_i$$

$$\frac{\partial W_1}{\partial S} = \sqrt{\frac{1\partial p}{S\rho\partial S}}, \frac{\partial W_1}{\partial u} = -1,$$

$$\frac{\partial W_2}{\partial S} = \sqrt{\frac{1\partial p}{S\rho\partial S}}, \frac{\partial W_2}{\partial u} = 1.$$

From this we obtain the total derivative for the characteristic variables:

$$dW_i = \sqrt{\frac{1\partial p}{S\rho\partial S}}.dS + (-1)^i du$$

and then, integrating, we get an explicit expression for them:

$$W_i = \int_{\hat{S}}^{S} \sqrt{\frac{1\partial p}{S\rho\partial S}} ds + (-1)^i u + C$$

The constant C can be taken equal to zero, based on the initial data: $W_i = 0$ for u = 0 and S = $\hat{S}$. Finally, we obtain: Conditions (1.6) are equivalent to the conditions:

$W_i(t) = g_i(t)$, (1.10)

where g_i (t) are given functions. As already mentioned, when solving system (1.1) at the boundary points of condition (1.10) along the characteristics leaving the region of integration, it is necessary to take into account (for i = 1 at the entrance to the vessel and for i = 2 at the exit from it). If we add to them the conditions (1.10) for the incoming characteristics (for i = 2 at the entrance to the vessel and for i = 1 at the exit from it), we get a correctly posed problem describing the blood flow in one vessel. Further in the dissertation, hemodynamics in the entire circulatory system is studied. In order to sew the solutions of equations (1.1) on different edges at the junction points of the vessels, we require the fulfillment of the laws Poiseuille and mass conservation:

$$p_k(S_k(t.x_k)) - p_{node}(t) = \varepsilon_k R_k S_k(t, x_k) u_k(t, x_k). \quad (1.11)$$

$$\sum_{i=1}^{K} \varepsilon_i S_i(t, x_k) u_i(t, x_k) = 0. \quad (1.12)$$

Here k = 1, ..., K, where K is the number of abutting vessels; pnode - pressure at the docking point; $\varepsilon_k = -1$ and $x_k = 0$ if the vessel leaves this point, and $\varepsilon_k = 1$ and $x_k = l_k$ otherwise (l_k is the length of the k-th vessel); R_k is the resistance of the vessel in this area. Consider the boundary conditions at the junction points of the vessels with the heart. Only one vessel enters / exits each chamber of the heart. Let the pressures at the ends of these vessels and in the corresponding chambers be equal. This condition, supplemented by a system of equations describing the work of the heart [8], sets the required set of boundary conditions. The arterial and venous parts of the circulatory system are connected through a network of arterioles, venules, and capillaries. The construction of a graph of vessels is impossible and unnecessary here. In addition, the sizes of these elements of the vascular network are comparable to the sizes of blood cells, so that the blood flow itself can not be described in terms of Newtonian fluid. For hemodynamic modeling, it is essential that the microvascular place creates hydrodynamic resistance, and, consequently, a pressure drop between arteries and veins. This pressure drop can be ensured by requiring the fulfillment of Poiseuille's law (1.11) with suitable values of the parameters at the interface between arteries and veins. Thus, in this work, the microvascular place is not described; instead, at the junction of arteries and veins, a standard system of boundary conditions (1.11) - (1.12) with suitable resistance values is used. In addition, to initialize the global circulation model, it is necessary to set the initial conditions. They can be chosen quite arbitrarily from the physiologically acceptable range, for example,

$S(0,x) = \hat{S}$ (1.13)

$u(0,x) = 0.$ (1.14)

Remark 1.1.1. This mathematical model is obtained directly from conservation laws written in integral form. However, it can be derived in other ways, for example, by integrating the Navier-Stokes equations over the cross section of the vessel [73]. This results in the following system:

$$\frac{\partial S}{\partial t}+\frac{\partial Q}{\partial x}=0,$$

$$\frac{\partial Q}{\partial t}+\alpha\frac{\partial}{\partial x}\left(\frac{Q^2}{S}\right)+\frac{S\partial p}{\rho\partial x}+K_r\left(\frac{Q}{S}\right)=0,$$

(1.15)

where Q = S_u is the fluid flow through the given section; α - coefficient correcting the pulse flux; K_r is a function describing friction.

Let us rewrite the second equation in (1.15) with α = 1 in the variables S and u:

$$\frac{\partial(Su)}{\partial t}+\frac{\partial(Su^2)}{\partial x}+\frac{S\partial p}{\rho\partial x}+K_r(u)=0.$$

(1.16)

Assuming that S, u $\in H^1([0, T] \times [0, l])$, we carry out equivalent transformations:

$$u\left(\frac{\partial S}{\partial t}+\frac{\partial Su}{\partial x}\right)+S\frac{\partial u}{\partial t}+Su\frac{\partial u}{\partial x}+\frac{S\partial p}{\rho\partial x}+K_r(u)=$$

(1.17)

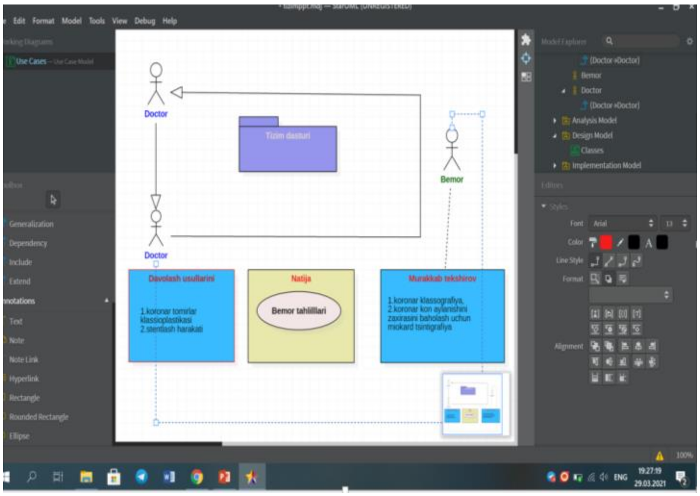
Considering the first equation (1.15), the expression in parentheses is zero. Divide the rest of the equality by S and put $\psi=-\frac{K_r(u)}{S}$

Under these conditions, the systems of equations (1.1) and (1.15) are equivalent.

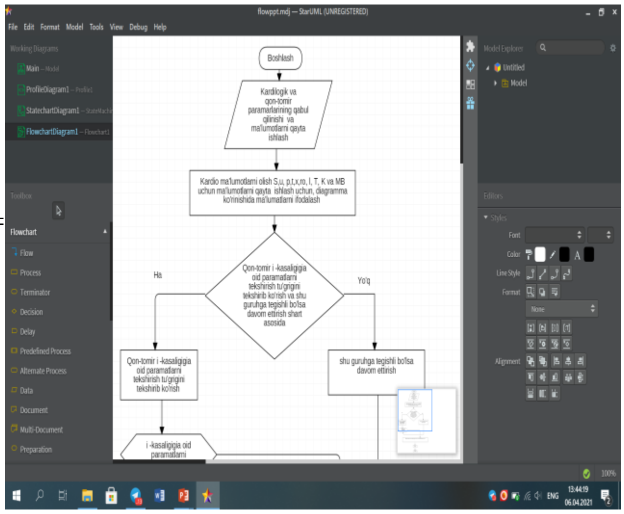
RESULTS AND CONCLUSION:

The described approach to constructing a numerical implementation makes it possible to divide the problem into independent blocks for calculating the flow in each vessel and at each point of their docking.

Although the described quasi-one-dimensional model of global circulation provides only averaged characteristics of blood flow, it is quite convenient to use, since it does not require large computational costs and, generally speaking, allows real-time calculations on computers with sufficient performance. The simplicity of the model makes it possible to complicate it and thereby take into account the influence of many factors.



Pic.1.1 Block Scheme of program UML



Pic.1.2. Algorithm of UML

Analysis of modern works on modeling blood flow using various averaging methods shows that such methods are capable of giving an adequate mathematical description of hemodynamics in a certain approximation. To achieve the goal, the following results are expected:

1. Analysis of the results of a healthy person and the general type of all cardiovascular diseases in a narrow area and fully averaged over space models allow to adequately describe the dynamics of cardiac output and integral parameters of blood flow throughout the body.

Stationary distributed models allow one to describe quasi-stationary pressure distributions in the microvascular place.

2. One-dimensional dynamic models successfully cope with the description of processes in the networks of large and medium vessels. Their integration with averaged and three-dimensional models makes it possible to simulate blood flow in a closed circulatory network, including hundreds and thousands of vascular segments.

REFERENCES:

1) Kholodov A. C., Simakov C. C. Numerical study of the oxygen content in human blood at low-frequency influences // Mathematical modeling. 2008.Vol. 20 (4). S. 87-102.
2) Kholodov A. C. Some dynamic models of external respiration and blood circulation, taking into account their connectivity and transport of substances// Computer models and progress of medicine / Ed. Belotserkovsky O. M., Kholodova A.S. Moscow: Nauka, 2001, pp. 127–163
3) Physiology of blood circulation: physiology of the vascular system / Edited by BL Tkachenko: Science, 1984.
4) Dyachenko A.I., Shabelkov V.G. Shabel'kov V. Mathematical models of the action of gravity on lung function. Moscow: Nauka, 1985.
5) Koshelev V.B., Mukhin S.I., Sosnin N.V., Favorsky A.P. Mathematical models of quasi-one-dimensional hemodynamics. M .: MAKS Press, 2010.
6) Sherwin S. J., Franke V., Peiro J., Parker K. One-dimensional modeling of vascular network in space-time variables // J. of Engineering Mathematics. 2003.V. 47. P. 217-250.
7) E. Faggiano, A. Antiga, G. Puppini, A. Quarteroni, G.B. Luciani, and C Vergara. Helical flows and asymmetry of blood jet in dilated ascending aorta with normally functioning bicuspid valve. Biomechanics and Modeling in Mechanobiology, 4 (12): 801–813, 2013.
8) E. Faggiano, T. Lorenzi, and A. Quarteroni. Metal artefact reduction in computed tomography images by a fourth-order total variation flow. Computer Methods in Biomechanics and Biomedical Engineering: Imaging & Visualization, 3-4 (4): 202–213, 20.