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USING AND MODELIZATION OF THE LAW THREE-DIMENSIONAL MODEL OF THE FLOW OF AN INCOMPRESSIBLE VISCOUS NEWTONIAN FLUID MODEL FOR BLOOD VESSEL WALLS

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ABSTRACT:

This article describes a mathematical model of the circulatory system for the cardiovascular system and provides a basic framework for the mathematical representation of cumulative medical parameters such as total vascular area about three-dimensional model of the flow of an incompressible viscous Newtonian fluid, blood volume, self-regulation, and effects on the upper and inner heart. In mathematical terms, linear dependencies, differential, integral and differential equations are used.
Keywords: linear dependence, integral-differential equations, logical-dynamic equations, general vascular zone, self-regulation, influence on the upper and working heart, medical parameters.

INTRODUCTION:

The tone of the mathematical model of the degree of relation to the cardiovascular system and the circulatory system of three-dimensional model of the flow of an incompressible viscous Newtonian fluid, which is related to time, is considered in the form of several specific areas of the cardiovascular system. When the experimental results, which reflect the volume tone and fluid dependence (blood fluidity), pressure and heart rhythm are reflected in the myocardial characteristics (in chronotope relations, dependence on velocity), the relationship between phases is shortened and weakened.

METHODS:

Consider the flow of a Newtonian incompressible viscous isothermal fluid in a limited volume with a fixed boundary. The standard model for describing such flows is the Navier-Stokes equations in the three-dimensional domain Ω. We will distinguish between three types of boundary conditions: entrance boundary Γ_{in} — Dirichlet condition on the solid boundary Γ_0 — the condition of adhesion on the outer boundary Γout is known for the normal component of the stress tensor:

$$\begin{cases} \rho\left(\frac{\delta_U}{\delta t} + (u \cdot \nabla)u\right) - \nu\Delta u + \nabla p = f \\ \qquad\qquad\qquad\qquad\qquad\qquad {}_B\Omega \\ div\, \boldsymbol{u} = 0 \\ \boldsymbol{u}|\Gamma_{in} = \boldsymbol{u}_{in}, \quad \boldsymbol{u}|\Gamma_0 = 0, \\ \left(\nu\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}} - p\boldsymbol{n}\right)|\Gamma_{out} = \phi, \\ \boldsymbol{u} = \boldsymbol{u_0}, t = t_0 \end{cases} \tag{1}$$

Here u is the velocity vector; p is the pressure; n is the normal vector to the surface; ρ — the density of the liquid; ν — the viscosity of the liquid. The functions f,, u_0 are known, t_0 — the initial moment of time. Next, we will assume that f = 0.

We will assume that at the input boundary Γ_{in}, the flow value averaged over time and space is positive, and at the output boundary out is negative. The conditions $u \cdot n < 0$ on Γ_{in} and $u \cdot n > 0$ on Γ_{out}, generally speaking, may not be fulfilled point wise. The condition for the normal stress at the outflow boundary is a natural boundary condition for the Navier-Stokes equations written in convective form. Its use is very effective for numerical calculations [1].

Suppose that the solution of system (1) is smooth. Then, by scalar multiplication of the equation of moments by the vector u and subsequent integration of the equality over the domain Ω, we obtain the following identity:

$$\frac{\rho}{2}\frac{d}{dt}||u||^2 + v\big||\nabla u||^2 + \int_{\Gamma_{out}\cup\Gamma_{in}} ((p + \frac{\rho}{2}|u|^2)I - v\nabla u)n \cdot uds = \int_{\Omega} f \cdot uds, \quad (1.1)$$

I – is the unit matrix. Here and further, $\|\cdot\|$ — L^2 is the norm. Let's denote the amount of energy for this problem*:*

$$\mathcal{E}_{3D} = \frac{\rho}{2}||u||^2$$

Since in our work f = 0, we rewrite the equality (1.1) in the following form:

$$\frac{d}{dt}\mathcal{E}_{3D} + v\|\nabla u\|^2 + \int_{\Gamma_{in}}((p + \frac{\rho}{2}|u|^2)\, I - v\nabla u\, n \cdot uds + \int_{\Gamma_{out}}(\frac{\rho}{2}|u|^2 n - \phi)uds = 0. \quad (1.2)$$

Integrating it in time, we get the following energy identity:

$$\mathcal{E}_{3D}(T) + v\int_0^T \|\nabla u\|^2\, dt + \int_0^T \int_{\Gamma_{in}} ((p + \frac{\rho}{2}|u|^2)I - v\nabla u)n \cdot udsdt + \int_0^T \int_{\Gamma_{out}} (\frac{\rho}{2}|u|^2 n - \phi) \cdot udsdt = \mathcal{E}_{3D}(0) \quad (1.3)$$

Under homogeneous conditions at all boundaries, equality (1.4) is equivalent

$$\frac{d}{dt}\mathcal{E}_{3D} + v\|\nabla u\|^2 + \int_{\Gamma_{out}} (\frac{\rho}{2}|u|^2 u \cdot nds = 0$$

to the assumption

$$\int_{\Gamma_{out}} |u|^2 u \cdot nds > 0 \quad \forall t > 0 \quad (1.4)$$

that $\frac{d}{dt}\mathcal{E}_{3D} \le 0$ takes place, and therefore, energy dissipation occurs in the three-dimensional model of the fluid flow (1.1). Condition (1.4) is often used as an assumption for analyzing blood flow models that include one-dimensional and three-dimensional models simultaneously (for example, [1, 2]), but in practice it is very difficult to verify it. The condition (1.4), in particular, is not true if reverse flows occur, as, for example, in the inferior vena cava [3, 4].

Assume that the approximation to the solution of the system of equations (1.1) $u_k\, p_k$ at time $t_k = k\Delta t$, $k = 1, \ldots, n$ designed and required to find the unknown u_{n+1}, p_{n+1}, with t_{n+1}. Approximating the time derivative with the second order accuracy in time t_{n+1}, we get the following scheme:

$$\begin{cases} \frac{1}{2\Delta t}(3\boldsymbol{u}^{n+1} - 4\boldsymbol{u}^n + \boldsymbol{u}^{n-1}) + w \cdot \nabla \boldsymbol{u}^{n+1} - v\nabla \boldsymbol{u}^{n+1} + \nabla \boldsymbol{p}^{n+1} = f^{n+1} \\ div\, \boldsymbol{u}^{n+1} = 0 \\ \boldsymbol{u}^{n+1}\big|\Gamma_{in} = \boldsymbol{u}_{in}^{n+1}, \ \ \boldsymbol{u}^{n+1}|\Gamma_{in} = 0, \quad \left(-v\frac{\partial \boldsymbol{u}^{n+1}}{\partial \boldsymbol{n}} + \rho^{n+1}\boldsymbol{n}\right)\Big|\Gamma_{out} = \phi^{n+1} \end{cases} \quad (1.5)$$

The choice of the expression for w allows either to linearize the convective term of the Navier-Stokes equations, or to preserve its nonlinearity. In the first case, the value of **w** is extrapolated from the solutions from the two previous time steps with the second order of accuracy:

$$\mathbf{w} = (2\,\mathbf{u}^{\,n} - \mathbf{u}^{\,n-1})\,, \quad (1.6)$$

thus, a linear system of differential equations is obtained, known as the Ozein problem. In the second case, the value of **w** is equal to the value of the speed at the *(n* + 1) thetime step:

$$\mathbf{w} = \mathbf{u}^{\,n+1}, \quad (1.7)$$

The problem (1.5) turns out to be nonlinear and we use the Newton-Krylov method to solve it.

RESULTS:

In all these cases, Newton's method converged in 2-4 iterations with a given absolute accuracy of 10-6, in 3-4 iterations with an accuracy of 10-8 to 10-12. The most significant effect on the number of iterations required to achieve a given accuracy is exerted by the value of the velocity in the adjacent vessels. More iterations are required when the pulse wave maximum passes through the node. This is due to a decrease in accuracy when choosing an initial approximation, since the solution on the upper time layer changes more intensively. Thus, the use of this approach is limited by the value of the maximum allowable flow through the node. All the computational experiments carried out have shown that this maximum lies far beyond the boundaries of physiologically correct values. Therefore, Newton's method is computationally efficient and convenient for this task.

CONCLUSION:

The described approach to constructing a numerical implementation makes it possible to divide the problem into independent blocks for calculating the flow in each vessel and at each point of their docking.

Although the described quasi-one-dimensional model of global circulation provides only averaged characteristics of blood flow, it is quite convenient to use, since it does not require large computational costs and, generally speaking, allows real-time calculations on computers with sufficient performance. The simplicity of the model makes it possible to complicate it and thereby take into account the influence of many factors.

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