NOVATEUR PUBLICATIONS
JournalNX- A Multidisciplinary Peer Reviewed Journal
ISSN No: 2581 - 4230
VOLUME 2, ISSUE 1, Jan-2016

DISCUSSION OF USUAL QUANTIZATION AND BRST QUANTIZATION, ALTERNATIVE QUANTIZATION

Safia Yasmin
Indas Mahavidyalaya, Indas, Bankura-722205, West Bengal, India
safiayasmin01513@gmail.com

ABSTRACT:
Discussed the phase space structure and theoretical spectrum of this model has been determined by Quantization named as Usual Quantization. BRST Quantization and alternative quantization also has been discussed. An alternative quantization of the gauge symmetric version of the model has been studied with Lorentz gauge to determine the phase space structure.

Keywords: BRST Quantization, Dirac Quantization, Alternative Quantization

INTRODUCTION:
A lagrangian is said to be singular when det[$\partial^2 L$
i j

] = 0, and that signifies the presence of constraint in the usual phase space [1, 2]. Constraint means velocity independent relation between coordinate and momentum [1, 2]. So all the velocities of the dynamical variables of a theory can not be determined in terms of momenta and as a result the precise canonical quantization gets threatened when a system contains constraints in its phase space. So quantization of this type of system is interesting in its own right and this type of quanti- zation is known as Dirac scheme of quantization of constraint system. This quantization has been done in the usual phase space [1, 2]. Through this quantiation it is possible to determine theoretical spectrum of a given model [1, 2].

An interesting extension based on Dirac method of quantization of constraint system is the well celebrated BRST formulation [3, 4, 5, 6, 7]. This formulation is applied to get the BRST invariant reformulation of lower dimensional models. BRST is a process to enlarge the phase space of a gauge theory and to restore the symmetry of the gauge fixed action in the extended phase space. But the process keeps the physical contents of the theory intact. It is an useful instrument to study unitarity and renormalizability of a given the- ory. Therefore we study the BRST invariant reformulations of a model [8, 9, 10] . BRST invariant reformulation of this model have been obtained using the Batalin, Fradkin and Vilkovisky (BFV)formalism [11, 12, 13, 14,15].The formalism of Batalin, Fradkin and Vilkovisky for quantization of a system is based on that a system with second class constraint is made first class in the extended phase space. The fields are needed for this transmutation, are converted into the Wess-Zumino scalar [16] with the appropriate choice of gauge condition.

In order to get BRST invariant action, auxiliary fields and ghost and anti-ghost fields are introduced. We also study the physical spectrum of the model using Dirac's scheme of quantization of constrained system.

In presence of Wess-Zumino term , alternative quantization is discussed here which is helpful to determine the canonical pair of fields which describe the Fock-space. The Lorentz type gauge fixing term at the action level is chosen for quantization in the alternative manner [5, 6, 7, 8, 9, 10]. Alternative quantization is the quantization in the extended phase space.

The paper is organied as follows the paper is organized as follows, in Section 2. phase space structure of the model has discussed through Dirac scheme of quantization. In Section 3. we briefly discuss BFV formalism and applying BFV formalism, how one can obtain the BRST invariant reformulation of this model. In sec 4 we have studied alternative quantization.

DISCUSSION OF QUANTIZATION IN THE USUAL PHASE SPACE:

In the usual phase space of theory, Dirac's scheme of quantization [1, 2, 3] plays a crucial role to determine the phase space structure of a theory. The canonical method of quantization requires the determination of momenta corresponding to the different field variables. As usual, A0 has no canonical conjugate, so there is a primary constraint π0 = 0. To preserve this constraint in time, it is necessary to have a further constraint, and this preservation may gives Gausss law. If the theory is anomaly free then no further constraint arises, and the above two constraints have vanishing Poisson brackets, i.e., are first class. In anomalous theories, two things occurs. If the constraints turn out to have nonvanishing Poisson brackets with among themselves, so that one always has second class constraints and there is no gauge invariance. In the other case, the closure of the set of second class constraints at the level of Gauss law indicates the occurrence of additional degrees of freedom. If the gauge current of the model is anomalous which leads to a gauge non-invariant structure. This quantization has been done in the usual phase space [1, 2]. Through this quantization it is possible to determine theoretical spectrum of a given model [1, 2].

When the model is not gauge invariant due to anomaly in the system. So, the study about the restoration of symmetry would be instructive which we have discussed in the next section using BFV formalism.

BRIEF DESCRIPTION OF BFV FORMALISM:

To make a theory BRST symmetric [3, 4, 5, 6, 7] we have to go through BFV formalism which consists of two steps. First step consists of converting the second class system to a first class system. Auxiliary fields are needed for this conversion. In the second step the ghost and anti-ghost fields are introduced. Some gauge fixing function are chosen. This allows one to define BRST charge and obtain BRST transformation of the fields. The unphysical ghost field is useful to bring back the symmetry of the gauge fixed action maintaining unitarity. This symmetry mixes all the fields (physical and ghost) in such a way that all the fields are treated in a same footing. But all the fields are forced to obey as different components in a single geometrical object.

Now we give the brief description of BFV formalism [11, 12]. Let us consider canonical Hamiltonian $H_c(q^i, p_i)$ and the constraints $w_i(q^i, p_i)$ are expressed in terms of canonical variables (q^i, p_i) in the phase space of a theory. Suppose that the constraints obey the following algebra

$[w_a, w_b] = iw_c U^c$, $[H_c, w_a] = iw_b V^a$, here U^c_{ab} and V^a_b are represented structure coefficients.

In order to extract out the physical degrees of freedom, the N number of additional condi- tions $\varphi^a = 0$ are introduced. These additional conditions $\varphi^a \approx 0$ choose in such a way that

we get $[\varphi^a, w_a] \neq 0$. φ^a play the role of gauge fixing functions. The constraints $\varphi^a = 0$ and $w_a = 0$ together with the Hamiltonian equations may be obtained from the action

$$S = \int [p_i\dot{q}^i - H(p_i, q^i) - \lambda^a w_a + \pi_a\varphi^a]dt, \quad (24)$$

where λ^a, π_a are the lagra

$i\delta^a_b$.

We introduce $(C^i, \bar{P}_i)$ and $(P^i, \bar{C}_i)$ satisfying the following algebra $[C_i, \bar{P}_i] = i\delta(x\text{-}y)$, $[P^i, \bar{C}_i] = i\delta(x-y)$ to make an equivalence to the initial theory. The partition function in the extended phase space which describe the quantum theory is given by

$$z_\psi = \int [dq_i dp_i d\lambda^a d\pi_a dC^a d\bar{P}_a dP^a d\bar{C}_a e^{iS} \quad (25)$$

The effective action in partition function's numerator is given as follows

$$S = \int [p_i\dot{q}^i + \bar{P}^i\dot{C}_i + \bar{C}^a\dot{P}_a - H_m + \dot{\lambda}^a\pi_a + i[Q, X]dt. \quad (26)$$

Here H_m is the minimal Hamiltonian, Q is the BRST charge and X is the gauge fixingfunction.

H_m is minimal Hamiltonian as termed by Batalin and Fradkin. It is given by the following

$$H_m = H_c + \bar{P}_a V^a C^b. \quad (27)$$

The $BRST$ charge Q and the fermionic gauge fixing function ψ are respectively given by

$$Q = C^a\omega - \frac{1}{2}C^bC U^c \bar{P}^a + P^a\pi \ ,(28)$$

$$\psi = \bar{C}_c\chi_a + \bar{P}^a\lambda^a,$$

where χ' s are expressed through the gauge fixing condition

$$\Phi_a = \dot{\lambda}^a + \chi_a.$$

We can get manifestly covariant action by choosing fermionic degrees of freedom φ properly. In order to show the equivalence between the BFV and the usual phase space quantization, the quantum effects related with the ghosts and the pure degrees of freedom, they mutuallycancel each other.

To make a theory BRST invariant we need to enlarge the Hilbert space of the theory to restore the symmetry in the gauge fixed action. The extended phase space includes auxiliary fields, ghost fields and anti-ghost fields in order to make BRST symmetric. There are various ways to get BRST invariant reformulation, BFV formalism is one of the them and the advantage of BFV formalism is already mentioned above. We get Wess Zumino [16] term in a transparent way, we use the Batalin, Fradkin and Vilkovisky (BFV) formalism. In fact one can apply developed version of FIK formalism to get BRST invariant effective action.

DISCUSSION ON ALTERNATIVE QUANTIZATION:

A gauge non invariant theory is made gauge invariant with the inclusion of Wess-Zumino [13] field in the extended phase space. In presence of Wess-Zumino term an attempt towards an alternative quantization is discussed here to determine the canonical pair of fields which describe the Fock-space. The Lorentz type gauge fixing term at the action level is chosen for quantization in the alternative manner. We conclude that, we get same spectrum through the alternative quantization as we got through the quantization in the usual phase space. This spectrum which is exact agreement with the mass term one can get through the Dirac scheme of quantization for gauge non invariant version of the same model in the usual phase space . Phase space needs to extended in order to restore the gauge symmetry. The fields needed for the extension will allocate themselves in the un-physical sector of the theory [6, 7].

CONCLUSION:

We have discussed quantization in the Usual Phase space. Through this quantization it is possible to determine theoretical spectrum of a given model [1, 2]. We have described the BRST invariant reformulation**.** In the BRST i**nvariant reformulations of the** models, extension of phase space have been needed because of the entry of the auxiliary fields in an essential way. The fields needed for the extension however keep **themselves** laid in the unphysical sector of the theory and the process keeps the physical content of the theory intact. Beauty as well as the advantage of this formalism is that the Wess Zumino terms appears automatically during the process of quantization. Note that the role of gauge fixing is very crucial to get the appropriate Wess-Zumino term in every case.

To establish the fact that the physical contents of the theory remains unaltered, alternative quantization of the gauge invariant models are needed out using Lorentz gauge [8, 9]. The spectrum suggests the appearance massive And massless boson. The extra equations appear because of the presence of the auxiliary field B in the Lorentz type gauge fixing term at the action level. The mass term of the massive boson remains the same in usual phase space and extended phase space too. It has found that the fields needed for the extension allocate themselves in the unphysical sector of the theory without hampering physical sector [6, 7].

REFERENCES

1) P.A.M. Dirac: Lectures on Quantum Mechanics. Yeshiva University Press, New York(1964)
2) A.J. Hanson, C. Teitelboim: Dirac General Method For Constrained Hamiltoniansystem
3) E. S. Fradkin, G. A. Vilkovisky: Phys. Lett. **B55** 224 (1975)
4) I. A. Batalin, E. S. Fradkin: Nucl. Phys. **B279** 514 (1987)
5) T. Fujiwara, I. Igarashi, J. Kubo : Nucl. Phys. **B314** 695 (1990)
6) I. A. Batalin, V. Tyutin: Int. J. Mod. Phys. **A6** 3255 (1991)
7) Y. W. Kim, S. K. Kim, W.T. Park, Y.J. Kim, K.Y. Kim : Phys. Rev. **D46** 4574 (1992)
8) A. Bassetto, L. Griguolo, P. Zanca: Phys. Rev. **D50** 1077 (1994)
9) A. Bassetto: Nucl. Phys. **B439** 327 (1995)
10) A. Bassetto, L. Griguolo, P. Zanca: Phys. Rev. **D50** 7638 (1994)
11) L. D. Faddeev: Phys. Lett. **B 154** 81 (1984)
12) L. D. Faddeev, S. L. Shatashvili: Phys. Lett. **B 167** 225 (1986)
13) S. Yasmin, A. Rahaman: Int.J Mod. Phys 31, 32 (2016)
14) A. Rahaman, S. Yasmin, Ann. Phys. 383, 497 (2017)
15) S. Yasmin, A. Rahaman: Int.J Theor. Phys 55 5172-5185 (2016)
16) J. Wess, B. Zumino: Phys. lett. **B37** 95 (1971)