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MATHEMATICAL ANALYSIS OF QUEUE WITH PHASE SERVICE

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ABSTRACT:

We have discussed a broad variety of topics, including phase service queueing models, in our conversations. In the realm of queueing theory, a number of models have been developed that include the concept of phased service. Several different types of daily and industrial situations have been tried to see whether these phase service queueing approaches are effective at alleviating congestion. This paper attempts to review the research done on phase service queues by the most eminent academics, as well as the study's relevance in a range of real-world queueing settings. Researchers' strategies for dealing with phase service queueing models have also been made public by a number of them. With the use of modelling and scientific techniques, we were able to organise all of the pertinent information. The primary audience for this work consists of system analysts, managers, and industry experts who are interested in applying queueing theory to analyse congestion situations when phase-type services are prevalent and who are interested in applying queueing theory to analyse congestion situations when phase-type services are prevalent.

INTRODUCTION:

A queue is simply a waiting line, and queues can be found in many aspects of our everyday lives, from the time we wake up to the time we go to sleep, from cashing checks or depositing money at the bank to the time we wait for a cell phone channel to open up at a base station. Queues can be found in many aspects of our everyday lives, from the time we wake up to the time we go to sleep, from the time we cash checks or deposit money at the bank to the time we It goes without saying that queues may be made up of either physical or immaterial objects. The occurrence of queues is caused by consumers' inability to find the service they have ordered when it arrives at the service facility. Customers are entities that need service, while servers are entities that provide that service in line with a specified set of rules, such as autos and petrol stations, among other things. Queuing theory, often known as queuing science, is the study of the act of waiting in a line. The queueing concept was first proposed by A.K. Erlang in 1909, and since then, this area of research has gone through a number of ups and downs in its development. As computer science has progressed, so has the spectrum of prospective applications for queueing systems research, which now includes anything from agriculture to astrophysics and everything in between. The ultimate objective of queueing system research is to establish a happy medium between the costs of waiting and the expenses of delivering service to customers.

One approach of minimising the length of time individuals spend in line in a queueing system is to increase the number of servers or the quantity of waiting space available. As a queueing system, the ultimate goal is to comprehend and improve the mechanism that operates across the system. It is the aim of this topic to examine queueing systems, including its many forms and uses in current life. Our study also focuses on a current examination of the relevant literature, which is at the heart of our findings. Depending on the size of the queue's buffer, you may have either an unlimited buffer capacity queueing system or a restricted buffer capacity queueing system in your queueing system.

(i) Infinite buffer capacity:

If we have an infinite buffer capacity, which is plausible when the arrival rate is greater than the service rate, the queue size will continue to grow indefinitely, eventually reaching an arbitrarily high size.

(ii) Finite buffer capacity:

When it comes to practical applications, the analysis of finite queueing systems is more beneficial than the study of infinite queueing systems. The greater the accuracy of the approximation, the greater the finite capability. An infinite capacity queueing system approximates the finite queueing system with a certain amount of safety margin, while the exact overflow probabilities for the finite capacity queueing model are the 'worst-case' results for the finite capacity queueing model.

1.2 CLASSIFICATION OF QUEUEING SYSTEM

Queueing systems may be categorised based on the probability distribution of arrivals and the service time distributions (i) Markovian queueing system (ii) Semi-Markovian queueing system and (iii) non-Markovian queueing system.

1.2.1 Markovian Queueing System

When the arrival rate is distributed according to a Poisson distribution and the service rate is distributed according to an exponential distribution, the Markovian queueing system is utilised. In the early stages of queueing theory, similar strategies were used in the telephone industry as well as in the computer industry. In such systems, the number of customers is represented by a birth-and-death Markov process, which is a discrete event.

$M/M/1$, $M/M/s$, $M/M/s/N$, $M/M/s/s$, $M/M/\infty$, $M^{[X]}/M/1$, $M/M^{[Y]}/1$.

1.2.2 Semi-Markovian Queueing System

The semi-Markovian queueing method is used when either arrival or service times do not match the exponential distribution.

$M/D/1$, $M/D/s$, $D/M/1$, $M/G/1$, $M/G/s$, $M/G/\infty$, $G/M/1$, $G/M/s$, $G^{[k]}/M/1$, $M^{[x]}/G/1$, $M/E_k/1$, $E_k/M/1$, $MAP/PH/1$, $PH/M/s$.

1.2.3 Non-Markovian Queueing System

The non-Markovian queueing system is one in which service times and arrival times are non-exponentially distributed.

$D/D/1$, $G/G/1$, $G/G/s$, $G/D/1$, $D/G/1$, $G/E_k/1$, $G/PH_k/1$, $E/E/1$, $E_j/E_k/s$, $Geo/G/1$, $GI/Geo/1$, $GI^X/Geo/s$.

1. MATHEMATICAL ANALYSIS OF QUEUE WITH PHASE SERVICE:

Quota-based queueing models with phase service are useful in illustrating and comprehending queueing issues in both everyday and industrial settings. We want to do this in this paper by providing a conceptual overview of phase service queueing models in a variety of different contexts. The principal service must be supplied to all incoming customers/jobs at the same time, according to traditional queueing models. The service may be delivered in a variety of phases using a variety of real-time technologies. Many academics have looked at the notion of optional phase services as a method to enhance the overall quality of the service provided. Occasionally, a small number of customers appear who need extra services in addition to the ones being provided. With phase service, queueing models are more accurate representations of real-world situations. The purpose of this work is to investigate queueing models with phase service and their applicability in real-world queueing problems. Queueing systems with phase service are being investigated due to the large number of various applications in performance evaluation and system dimensioning, such as industrial systems, information and communication networks (ICNs), inventories, and supply chains. Over the past few decades, academics have been more interested in phase service queueing models, which are described below. This issue has seen significant transformation throughout the years, and more research is required in this field. Between the 1980s and 1990s, there was a flurry of research on phase service queueing models that appeared in the literature.

It is our hope that the many applications of phase service queueing models in a number of settings will help a wide range of everyday and industrial situations, which is the driving force for our study into these models in a variety of contexts. For the sake of illustration, we'll utilise a production system with a number of workstations to demonstrate our argument. Server Waiting List Phase Service is a paradigm for an industrial machine that generates a wide range of items in a single operation. Following that, we'll take a look at how the Internet and the VoIP protocol interact with one another. People may have low- or no-cost phone calls via an IP network, such as the internet, by utilising VoIP, which is a new and highly sought-after technology that is still in its early stages. In VoIP protocols, the majority of the service is dependent on the transmission of caller information from one end user to another. The VoIP service is comprised of three steps: the establishment of the connection, the transfer of information, and the accessing of the service. It was the notion of a phase service queue that came about because certain computer and communication networks utilise a message producing centre that functions as both a server and a pacing box to process messages in phases, which gave origin to the term "phase service queue."

2. PHASE SERVICE SYSTEM:

Researchers in the domains of queueing theory and reliability theory have given considerable attention to queueing systems with phase service because of its inherent dependability. Many well-known academics have conducted substantial research into phase service queueing models in order to enhance the service grade as a result of their widespread use in a variety of industrial queueing issues sectors. Services from a range of different providers have an influence on the real-time system. Using a phase service queueing system, all customers are served in a sequential manner; some of these are required phases of service given by the server to all customers, while others are optional services offered by the server to consumers depending on their choices.

The exponential and Poisson distributions are the most often used distributions in real-time systems because of their simplicity. As a result of the fact that queueing models with exponential distributions are practical and easy to handle, they have become more popular. Making an analytical model based on an accurate approximation of real-world systems is the fundamental purpose of determining and estimating the parameters of an acceptable distribution when selecting and estimating its parameters. We may mitigate some of the disadvantages of exponential distributions by mixing them into a more sophisticated distribution. The service is made available in phases. Some popular distributions for exhibiting phase-type service are shown in the following table.

3.1 Phase Type Distribution:

This distribution is an example of the phase-type distribution, which also contains the distributions shown in the following instances. It is possible to organise single-phase distributions in several ways, including sequential and parallel setups. The distribution of phase types is represented by the letter "PH" in the symbol. An example of this distribution is represented by Markov chains with states (also known as phases) and a transient transition probability matrix. If the random variable's phase-type distribution spans the whole time interval between the beginning of the Markov chain and the conclusion of the Markov chain, the random variable has a phase-type distribution. On the positive real axis, in particular, we may refer to phase type as the absorption time distribution of a specific Markov jump process, which seems to be a good compromise between generality and tractability when applied to Markov jump processes in general. For more information, we refer you to the work of Neuts and Asmussen.

3.2 Two-Phase Essential Service:

Two-phase essential service is provided. Queueing models are queueing models in which the server offers two phases of vital service to each client in order for the client to complete the transaction. In many real-time systems, such as manufacturing systems, circumstances like this might arise, where a machine that is producing a specific item may need two steps of care to be performed consecutively on the object. For raw materials to be processed completely, the first stage of service (periodic inspection) must be completed before moving on to the second stage of service (regular processing). If just one server is used to deliver first and second essential services (FES and SES), Kumar and Arumuganathan assumed that the server was responsible for all incoming calls before providing first and second essential services (FES and SES).

3.3 Two-Phase Optional Service:

As a practical matter, many vocations need both the second optional service and the first compulsory service in order to be able to do their duties successfully (FES). As part of his investigation, Madan looked at the possibility of a second, entirely optional obligation. A server in the M/G/1 paradigm provides first-level required services to all incoming customers, and then a small fraction of those customers may request a second level of optional services from the server on their behalf (SOS). Customers may come to a cyber cafe to browse the Internet (as FES), but they may also need the scanning of certain files (as SOS) while they are at the establishment.

3.4 Multiphase Essential Service:

When the server is serving the consumers in a queueing system, there are only a limited number of stages to go through. Each customer is required to complete all of these stages in order to complete the service process. Various examples of this phenomenon include the clinical physical examination procedure, which includes a physical examination of the ear, nose, and throat, blood tests, electrocardiograms, and eye exams; all, which are considered essential phases of service by any medical doctor before they can declare a patient fit for further treatment. A batch arrival queueing system with a modified Bernoulli vacation policy was recently investigated in the context of a customer who requires successive stages of service, i.e. first stage service followed by stage two (SSS), stage two followed by stage three (TSS), and so on until stages of service (TSS) are provided.

3.5 Multiphase Optional Service:

In multioptional phase service queueing models, the first essential service (FES) is delivered to all arriving customers as a matter of course. After the completion of the FES, the server provides a second optional service (SOS) to the client. If no request is received, the server departs the system with a probability proportional to the number of requests received. Once SOS has been performed, the customer may select a third optional service, or they may choose to exit the system. Customers have the choice of abandoning the system or requesting any of the phase optional services with equal chances of success. Consumers often come to "Malls" in search of a movie, and they are less likely to request other attractions such as shopping, kid's activities, or eating out when they do. Unreliable server batch arrival queueing system with important and multi-optional services under policy was addressed by Jain and Upadhyaya, respectively, in their contributions to the journal They employed the probability generating function technique, for example, to get the system size distribution and other performance measures, among other things.

3. QUEUEING SYSTEM WITH AN UNRELIABLE SERVER:

The regulation of queues is one of the most researched areas of the queueing problem. A controllable queueing model attempts to discover the most cost-effective method of turning on or off the server, so that a decision-maker may do it at the lowest feasible cost. A thorough assessment of the factors that may be controlled in lineups is provided by Tadj and Choudhury in their paper. It is possible to categorise previous work on managed lineups into two categories: service management and arrival management. Yadin and Naor first presented the N-policy M/M/1 queueing system without startup in the context of regulating services, and it has since gained widespread acceptance. A r-quorum queueing system driven by N-policy was investigated by Tadj, who used matrix analytic techniques to investigate the system. Baker was the first to propose an M/M/1 queueing system with N policies and exponential beginning durations, which was later adopted by the industry. It was determined that the general beginning time of Baker's model needed to be prolonged for Borthakur et al.

It's simple to notice the parallels between queueing models that use an unstable server and real-world circumstances that are presented here. It is possible that the server may have unforeseen faults while providing service. By Wang et al., the steady-state solutions of the N-policy M/M/1, the N-policy M/Ek/1, and the N-policy M/H2/1 queueing systems have been found. This is the first time that this has been done for a queueing system. Wang et al. extended the Npolicy M/HK/1 queueing approach by making it more generic. Ke and Pearn devised closed-form solutions for the N-policy M/M/1 queue with server breakdowns and repeated vacations, which they called the M/M queue with server breakdowns and repeated vacations. Wang and Ke computed the expected number of customers in the three-control policy-controlled M/G/1 queueing system with server failures using a policy control M/G/1 queueing system with three controls.

Typically, before a service can be launched, the server must first be started and configured. Numerous authors, including Lee and Park, Medhi and Templeton, Takagi, and others, have investigated the commencement times of M/G/1 queues with N-policy policies. Ke recently examined various essential system parameters, such as server downtime and startup/breakdown times, in order to get a deeper understanding of the N-policy M/G/1 queue. Wang et al maximum entropy's approach was used to estimate the length distribution of an N-policy M/G/1 queue with server breakdown and start-up queue length distribution. Gupta and Melachrinoudis observed complimentary correlations between N-policy and F-policy using finite source queueing models with spares, which they used in their research. The connection between the N- and F-policies is established via a series of assumptions made by Gupta. According to Karaesmen and Gupta, the stationary queue length distributions for the two queueing systems under N-policies and F-policies were determined utilising the duality relationship for the two queueing systems. Wang et al. have investigated the optimal management problem for M/G/1/K and G/M/1/1/K queueing systems with integrated F-policy and exponential starting time.

4. CONCLUSION:

For the purpose of this paper, we've summarised the current state of the research in queueing systems with phase service. Using phase service queueing models, researchers have been able to model and investigate a wide variety of daily and industrial processes. This research, which takes a queue-theoretic approach, tries to give a coherent framework for investigating phase service models by combining many existing studies. Research has been conducted on phase service queueing models, which are a combination of numerous concepts. To represent traffic congestion more realistically, system analysts, engineers and managers prefer queueing models with phase service. This is because queueing models with phase service assist in solving the problem of congestion and can be used as an effective tool to reduce system blockage and delay. It was decided to employ an actual-world situation in order to recreate the aforementioned impact.

REFERENCES:

1. M. Jain and P. K. Agrawal, “M/Ek/1 Queueing system with working vacation,” Quality Technology & Quantitative Management, vol. 4, no. 4, pp. 455–470, 2007.
2. K.-H. Wang, H.-T. Kao, and G. Chen, “Optimal management of a removable and non-reliable server in an infinite and a finite $M/Hk/1$ queueing system,” Quality Technology & Quantitative Management, vol. 1, no. 2, pp. 325–339, 2004.
3. K. H. Wang and K. L. Yen, “ptimal control of an M/Hk/1 queueing system with a removable server,” Mathematical Methods of Operations Research, vol. 57, pp. 255–262, 2003.
4. M. S. Kumar and R. Arumuganathan, “On the single server batch arrival retrial queue with general vacation time under Bernoulli schedule and two phases of heterogeneous service,” Quality Technology & Quantitative Management, vol. 5, no. 2, pp. 145–160, 2008.
5. J. A. Zhao, B. Li, X. R. Cao, and I. Ahmad, “A matrix-analytic solution for the DB MAP/PH/1 priority queue,” Queueing System, vol. 53, pp. 127–145, 2006.
6. R. Sharma and G. Kumar, “Unreliable server M/M/1 queue with priority queueing system,” International Journal of Engineering and Technical Research, pp. 368–371, 2014.
7. J. Wu, Z. Liu, and G. Yang, “Analysis of the finite source MAP/PH/N retrial G-queue operating in a random environment,” Applied Mathematical Modelling, vol. 35, no. 3, pp. 1184– 1193, 2011.
8. G. Choudhury and M. Paul, “A two phase queueing system with Bernoulli feedback,” International Journal of Information and Management Sciences, vol. 16, no. 1, pp. 35–52, 2005.

9. U. Krieger, V. I. Klimenok, A. V. Kazimirsky, L. Breuer, and A. N. Dudin, "A $BMAP \mid PH \mid 1$ queue with feedback operating in a random environment," Mathematical and Computer Modelling, vol. 41, no. 8-9, pp. 867–882, 2005.
10. G. Choudhury and K. C. Madan, "A two-stage batch arrival queueing system with a modified Bernoulli schedule vacation under N-policy," Mathematical and Computer Modelling, vol. 42, no. 1-2, pp. 71–85, 2005.