MAPLE PROGRAM TO THE SOLUTION OF EQUATIONS REPRESENTING PROBLEMS OF HEAT DISPOSION

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Annotation. This paper shows the ways of solving partial differential equations describing heat conduction problems using the Maple system .

Keywords: heat conduction, differential equation, problem, rod, initial condition, boundary condition, Maple.

It is known that heat generation, vibration problems are dynamic problems, which are often expressed by partial differential or integro-differential equations. An example of the simplest problem of heat dissipation is the problem of heat dissipation on a rod, which is represented by partial derivatives of parabolic differential equations. Several analytical methods for solving problems of heat dissipation on a rod are given in textbooks. But the formulations of the solutions obtained by modern computers and software are not shown. One of the ways to solve such problems is the use of the Maple mathematical system [5,7]. Therefore, here we will consider the solution of some problems of heat dissipation on the rod using the Maple mathematical system and how to use it in the course of the lesson.

Let's look at the process of heat dissipation in Avalon, a semi-constrained rod. To do this, use this equation $\frac{\partial}{\partial t}u(t,x)=a^2\left(\frac{\partial^2}{\partial x^2}u(t,x)\right)$

the following initial condition $u(0,x)=f(x)$, and with these boundary conditions we look at:

1. The limit of the rod $x=0$ does not conduct heat at the point: $\frac{\partial}{\partial t}u(t,0)=0$ yoki

2. Sturgeon chegarasini $x=0$ the same temperature is maintained at the point: $u(t,0)=T_0$.

Here is a sequence of commands for solving the problem in the Maple system and the resulting one.

> restart ;

The expression of the equation and its solution by the method of separation of variables:

> PDE:=diff(u(t,x),t)=a^2*diff(u(t,x), x,x);
struc := pdsolve (PDE,HINT=T(t)*X(x));

$$PDE := \frac{\partial}{\partial t}u(t,x)=a^2\left(\frac{\partial^2}{\partial x^2}u(t,x)\right)$$

$$struc := (u(t,x) = T(t)X(x)) \wp \, where \left[\left\{\frac{\partial}{\partial t}T(t) = \_c_1 T(t), \frac{\partial^2}{\partial x^2}X(x) = \frac{\_cX(x)}{a^2}\right\}\right]$$

> dsolve(diff(T(t),t)=_c[1]*T(t));
dsolve (diff(X(x),`$`(x,2))=_c[1]*X(x)/a^2);

$T(t) = \_C1e^{(\_c_1 t)}$

$X(x) = \_C1e^{\left(\frac{\sqrt{\_c_1}x}{a}\right)} + \_C2e^{\left(\frac{\sqrt{\_c_1}x}{a}\right)}$

Doing replacement : $\_c_1$ = = - λ^2

> dsolve (diff(T(t),t)=-lambda^2*T(t)*a^2);
dsolve (diff(X(x),`$`(x,2))=-lambda^2*X(x));

$T(t) = \_C1e^{\left(-\lambda^2 a^2 t\right)}$

$X(x) = \_C1\sin(\lambda x) + \_C2\cos(\lambda x)$

Imagine general solution :
> u(t,x):=(C1*sin(lambda*x)+C2*cos(lambda*x))*exp(-lambda^2*a^2*t);

$u(t,x) := (C1\sin(\lambda x) + C2\cos(\lambda x))e^{\left(-\lambda^2 a^2 t\right)}$

$u(t,x)$ - the function is optional for the given uesiimi equation (where is the variable parameter in the range of values from D o), but for each λ it is suitable $C1(\lambda)$ va $C2(\lambda)$ coefficients match.
Therefore, we write like this: > u [lambda](t , x):=(C 1(lambda)* sin (lambda * x)+ C 2(lambda)* cos (lambda * x))* exp (- lambda ^2 * a ^2* t);

$$u_\lambda(t,x) := (C1(\lambda)\sin(\lambda x) + C2(\lambda)\cos(\lambda x))e^{\left(-\lambda^2 a^2 t\right)}$$

Received y e chim the result λ param e trga it 's like the sup e position of all grasses _ in the form we express :
>u(t,x):=int(u[lambda](t,x), lambda=-infinity..infinity);

$$u(t,x) := \int_{-\infty}^{\infty}(C1(\lambda)\sin(\lambda x) + C2(\lambda)\cos(\lambda x))e^{\left(-\lambda^2 a^2 t\right)}d\lambda$$

$C1(\lambda)$ and $C2(2)$ coefficient e nts look at it for start drinking _ from the conditions we use :
> u_ 0(t,x):= eval (subs(t=0, u(t,x)))=f(x);

$$u\_0(t,x) = \int_{-\infty}^{\infty} C1(\lambda)\sin(\lambda x) + C2(\lambda)\cos(\lambda x)d\lambda = f(x)$$

This is an expression $f(x)$ function Fur e to the int e gral spread with fits : _ _ _
> f(x)=(1/(2*Pi))*int(int(f(xi)*cos(lambda*(xi-x)),xi=-infinity..infinity),lambda=-infinity. .infinity);

$$f(x) = \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda(\xi - x))d\xi d\lambda\right)$$

Demak, $C1(\lambda)$ va $C2(\lambda)$ koeffisientlar quyidagicha bo`ladi:
> C1(lambda):=(1/(2*Pi))*int(f(xi)*sin(lambda*xi),xi=-infinity..infinity);
>C2(lambda):=(1/(2*Pi))*int(f(xi)*cos(lambda*xi),xi=-infinity..infinity);

$$C1(\lambda) := \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty} f(\xi)\sin(\lambda\xi)d\xi\right)$$

$$C2(\lambda) := \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda\xi)d\xi\right)$$

>u(t,x):=int((C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-lambda^2*a^2*t),lambda = -infinity .. infinity);
>u(t,x):=combine(int((C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-lambda^2*a^2*t),lambda = -infinity .. infinity));

$$u(t,x) := \int_{-\infty}^{\infty}\left(\frac{1}{2}\left(\frac{\sin(\lambda x)}{\pi}\int_{-\infty}^{\infty} f(\xi)\sin(\lambda\xi)d\xi\right)+\frac{1}{2}\left(\frac{\cos(\lambda x)}{\pi}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda\xi)d\xi\right)\right)e^{\left(-\lambda^2a^2t\right)}d\lambda$$

$$u(t,x) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{2}\frac{e^{\left(-\lambda^2a^2t\right)}f(\xi)\cos(-\lambda x+\lambda\xi)}{\pi}d\xi d\lambda$$

The resulting expression can also be expressed in different ways. To do this, consider the following integral:
> int (exp (-lambda^2*a^2*t)* cos (-lambda* x+lambda *xi), lambda = -infinity .. infinity);

$$\int_{-\infty}^{\infty} e^{\left(-\lambda^2a^2t\right)}\cos(-\lambda x+\lambda\xi)d\lambda$$

We substitute the variables and substitute the function under the Integral :
> Simplify(subs({xi=-v*a*t^(1/2)+x,lambda=w/(a*sqrt(t))},exp(-lambda^2*a^2*t) *cos(-lambda*x+lambda*xi)));

$$e^{\left(-w^2\right)}\cos(wv)$$

> Int (exp (-lambda^2*a^2*t)* cos (-lambda* x+lambda *xi),lambda = -infinity .. infinity)=(1/(a* sqrt (t))) * int (exp (-w^2)* cos (w*v),w = -infinity .. infinity);

$$\int_{-\infty}^{\infty} e^{\left(-\lambda^2a^2t\right)}\cos(-\lambda x+\lambda\xi)d\lambda = \frac{\sqrt{\pi}e^{\left(-\frac{v^2}{4}\right)}}{a\sqrt{t}}$$

> Int(exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi),lambda=-infinity..infinity)= subs(v=(x-xi)/a/t^(1/2),1/a/t^(1/2)*Pi^(1/2)* exp (-1/4*v^2));

$$\int_{-\infty}^{\infty} e^{\left(-\lambda^2a^2t\right)}\cos(-\lambda x+\lambda\xi)d\lambda = \frac{\sqrt{\pi}e^{\left(-\frac{(x-\xi)^2}{4a^2t}\right)}}{a\sqrt{t}}$$

Received expression solutions will be :
>u(t,x):=(1/(2*a*sqrt(Pi*t)))*int(f(xi)*exp(-1/4*(x-xi)^2/a^ 2/t),xi = -infinity .. infinity);

$$u(t,x) := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty} f(\xi)e^{\left(-\frac{(x-\xi)^2}{4a^2t}\right)}d\xi\right)$$

Now consider the above boundary conditions .
1 . The rod $x=0$ tip at the point does not allow heat to pass through, i.e., it is insulated:

$\frac{\partial}{\partial x}u(t,0)=0$.

In this case, $f(x)$ it is necessary to continue in pairs with the negative semiaxis:

$f(x)=f(-x)$

>u_x(t,x):=diff(u(t,x),x);

$$u\_x(t,x):=\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty}-\frac{1}{2}\frac{f(\xi)(-\xi+x)e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}}{a^2t}d\xi\right)$$

We substitute a variable :

> u0_x:=subs(x=0 ,u _x(t,x));

$$u0\_x:=\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty}\frac{1}{2}\frac{f(\xi)\xi e^{\left(-\frac{\xi^2}{4a^2t}\right)}}{a^2t}d\xi\right)$$

Here the function under the integral is odd; therefore, the Integral is equal to zero, and the boundary condition is satisfied. Therefore, the solution can be written as:

> u(t,x):=1/2*1/a/(Pi*t)^(1/2)*int(f(xi)*(exp(-1/4*(-xi+x)^2/a^2/t)+exp(-1/4*(xi+x)^2/a^2/t)),xi=-infinity..infinity)

$$u(t,x):=\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty}f(\xi)\left(e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}+e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}\right)d\xi\right)$$

2 . The rod $x=0$ let the temperature at the limit is constant $(0<x)$ that is: $u(t,0)=T_0$

In this case, substituting, we get the boundary condition homogeneous:

> U(t,x)=u(t,x)-T0;

> F(x)=f(x)-T0;

$U(t,x)=u(t,x)-T0$

$F(x)=f(x)-T0$

After that, $F(x)$ you need to continue based on the negative semi-accuracy of the point: $F(x)=-F(-x)$.

The solution to the problem looks like this:

> U(t,x):=1/2*1/a/(Pi*t)^(1/2)*int(F(xi)*exp(-1/4*(-xi+x)^2/a^2/t),xi = -infinity .. infinity);

$$U(t,x):=\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty}F(\xi)e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}d\xi\right)$$

Заменяем местами:

> F(xi):=f(xi)-T0;

>u(t,x):=T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi)+T0)*exp(-1/4*(-xi+x)^2/a^2/t),xi = -infinity .. infinity);

$F(\xi):=f(\xi)-T0$

$$u(t,x):=T0+\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty}(f(\xi)+T0)e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}d\xi\right)$$

$F(\xi):=f(\xi)+T0$ учитываем нечетность функции:

> u(t,x):=T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi)-T0)*exp(-1/4*(-xi+x)^2/a^2/t),xi = -infinity .. 0)+1/2*1/a/(Pi*t)^(1/2)*int((f(xi)-T0)*exp(-1/4*(xi+x)^2/a^2/t),xi = 0 .. infinity);

$$u(t,x) := T0+\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty}\left(f(\xi)+T0\right)e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}d\xi\right)+\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty}\left(f(\xi)-T0\right)e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}d\xi\right)$$

Или

> u(t,x):=T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi))*(exp(-1/4*(- xi+x)^2/a^2/t)-exp(-1/4*(xi+x)^2/a^2/t)),xi=0..infinity)-T0*Integr;

$$u(t,x) := T0+\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty} f(\xi)e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}-e^{\left(-\frac{(\xi+x)^2}{4a^2t}\right)}d\xi\right)-T0Integr$$

here :

> Integr:=1/2/a/(Pi*t)^(1/2)*Int((T0)*exp(-1/4*(-xi+x)^2/a^2/t) ,xi = -infinity .. 0)-1/2/a/(Pi*t)^(1/2)*Int((T0)*exp(-1/4*(xi+x)^2/a ^2/t),xi = 0 .. infinity);

$$Integr := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{0} T0e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}d\xi\right)-\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{0}^{\infty} T0e^{\left(-\frac{(\xi+x)^2}{4a^2t}\right)}d\xi\right)$$

To calculate the integrals, we make the following substitutions: $\xi = x-2at^{\left(\frac{1}{2}\right)}y, \xi = -x+2at^{\left(\frac{1}{2}\right)}z.$

> subs(xi=x-2*a*t^(1/2)* y,exp (-1/4*(- xi+x)^2/a^2/t));
> subs(xi=-x+2*a*t^(1/2)* y,exp (-1/4*(xi+x)^2/a^2/t));

$$e^{(-y^2)}$$

$$e^{(y^2)}$$

> I1:=simplify((1/a/(Pi*t)^(1/2))*int(exp(-y^2),y = -infinity .. x/(2*a*t^(1/2)))*2*a*t^(1/2))/2;
>I2:=simplify((1/a/(Pi*t)^(1/2))*int(exp(-y^2),y = x/(2*a*t^(1/2)) ..infinity)*2*a*t^(1/2))/2;

$$I1 := \frac{1}{2}+\frac{1}{2}erf\left(\frac{x}{2a\sqrt{t}}\right)$$

$$I2 := -\frac{1}{2}erf\left(\frac{x}{2a\sqrt{t}}\right)+\frac{1}{2}$$

>Integr:=simplify(I1-I2);

$$Integr := erf\left(\frac{x}{2a\sqrt{t}}\right)$$

Таким образом, мы получаем:

> u(t,x):=collect(T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi))*(exp(-1/4*(-xi+x)^2/a^2/t)-exp(-1/4*(xi+x)^2/a^2/t)),xi=0..infinity)-T0*Integr,T0);

$$u(t,x) := \left(-erf\left(\frac{x}{2a\sqrt{t}}\right)+1\right)T0+\frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty} f(\xi)e^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}-e^{\left(-\frac{(\xi+x)^2}{4a^2t}\right)}\right)d\xi$$

Пример.

> restart;

The following one-sex equation

$\frac{\partial}{\partial t}u(t,x)=a^2\left(\frac{\partial^2}{\partial x^2}u(t,x)\right)$

this is boundary condition :

$$\frac{\partial}{\partial x}u(t,0)=0$$

and this is the initial condition $u(0,x)= f(x)$,

solve with, here's $f(x)$ the function looks like this:

```
> a :=1;l:=4;L:=6;alpha:=1;
>f(x):=x->piecewise(x<l ,0 , x< L,alpha , x>L,0);
```

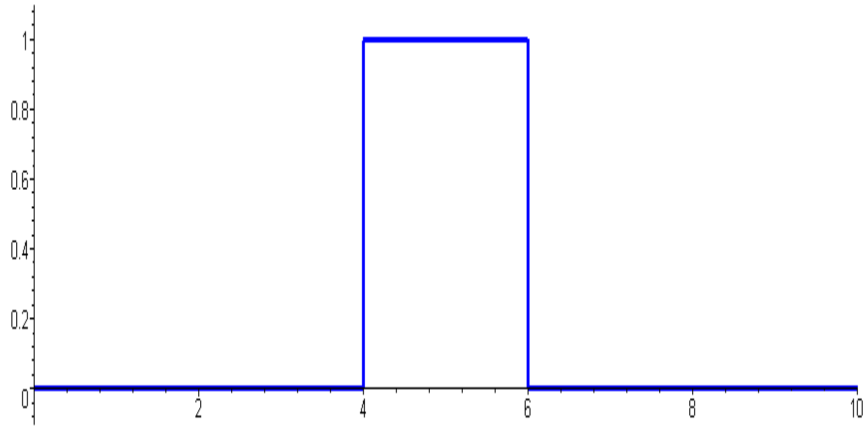
$a:=1$

$l:=4$

$L:=6$

$\alpha:=1$

```
> plot( f(x),0..10,-0.1..1.1, numpoints =400,color= blue,thickness =3);
```




```
> restart;
>f(xi):=xi->piecewise(xi<l,0, xi<L,alpha, xi>L,0);
```

Формула решения задачи:

```
> u(t,x):=simplify(1/2*1/a/(Pi*t)^(1/2)*int(f(xi)*(exp(-1/4*(-xi+x)^2/a^2/t)+exp(-1/4*(xi+x)^2/a^2/t)),xi = l .. L));
```

$u(t,x):=-\frac{1}{2}c\left(erf\left(\frac{l-x}{2a\sqrt{t}}\right)+erf\left(\frac{l+x}{2a\sqrt{t}}\right)-erf\left(\frac{L-x}{2a\sqrt{t}}\right)-erf\left(\frac{L+x}{2a\sqrt{t}}\right)\right)$

```
> a:=1;l:=4;L:=6;alpha:=1;
```

$a:=1$

$l:=4$

$L:=6$

$\alpha:=1$

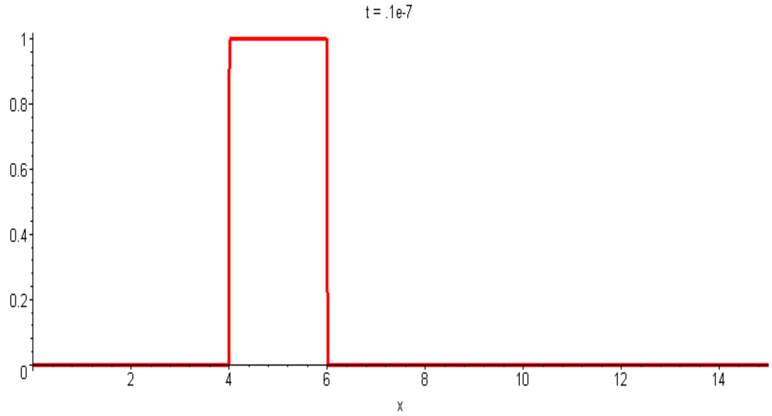
Решение уравнения:

```
> with(plots):
>u(t,x):=-1/2*(erf(1/2*(l-x)/a/t^(1/2))+erf(1/2*(l+x)/a/t^(1/2))+erf(1/2*(-L+x)/a/t^(1/2))-erf(1/2*(L+x)/a/t^(1/2)));
```

$u(t,x):=-\frac{1}{2}erf\left(\frac{4-x}{2\sqrt{t}}\right)-\frac{1}{2}erf\left(\frac{4+x}{2\sqrt{t}}\right)-\frac{1}{2}erf\left(\frac{-6+x}{2\sqrt{t}}\right)+erf\left(\frac{6+x}{2\sqrt{t}}\right)$

Let's draw a two-dimensional animated graph of the obtained solution:

> animate(plot,[u(t,x),x=0..15], t=0.00000001..12, frames=60,thickness=3);



Here are the graphs of the obtained solution for several points in time:

> tau :=12:

u_1(x):=subs(t=tau*0.000001,u(t,x)):

u_2(x):= subs(t=tau*(1/8),u(t,x)):

……………………………………………………

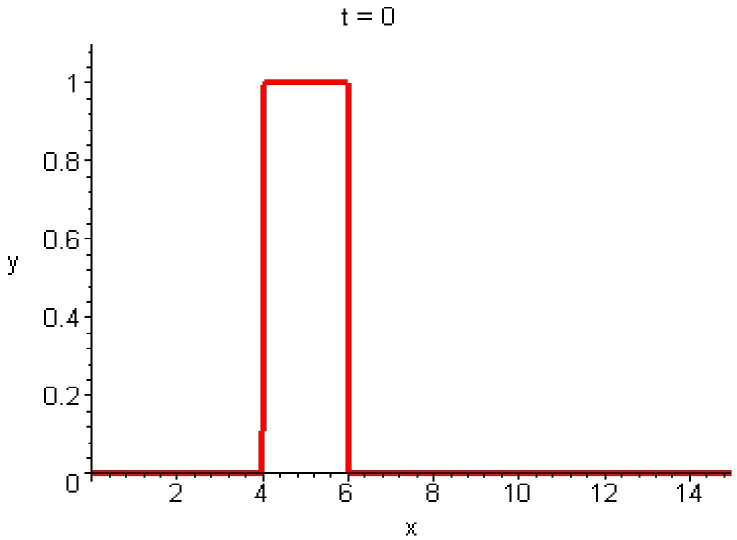
u_8(x):= subs(t=tau*(7/8),u(t,x)):

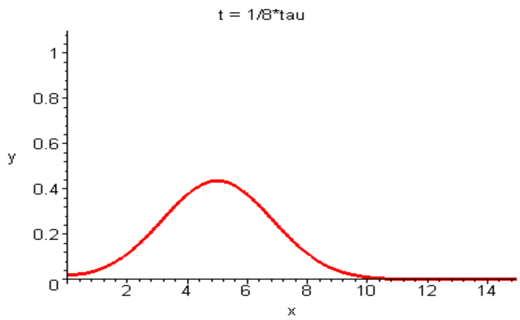
plot(u_1(x),x=0..15,y=-0.02..1.1,title="t = 0", color=red,thickness=3);

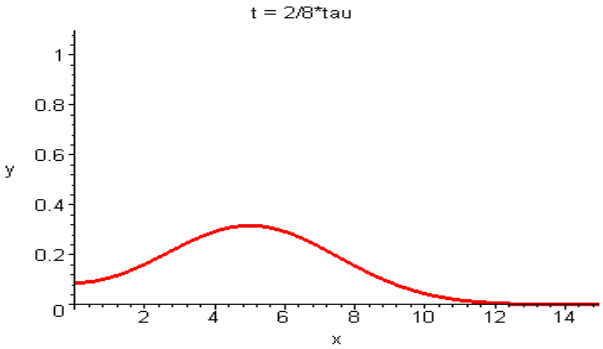
plot(u_2(x),x=0..15,y=-0.02..1.1,title="t = 1/8*tau",color=red,thickness=3);

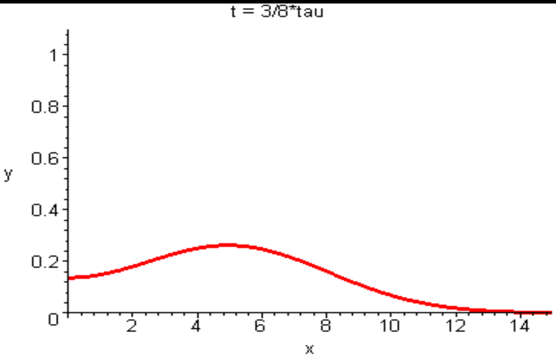
………………………………………………………………………………

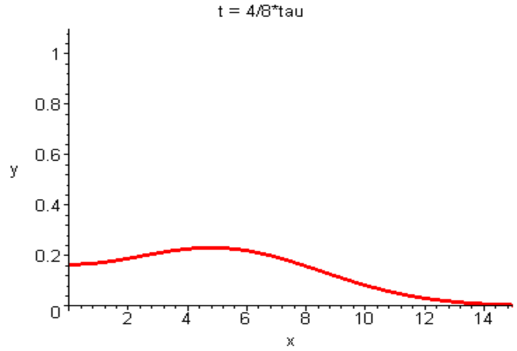
plot(u_8(x),x=0..15,y=-0.02..1.1,title="t = 7/8*tau",color=red,thickness=3);

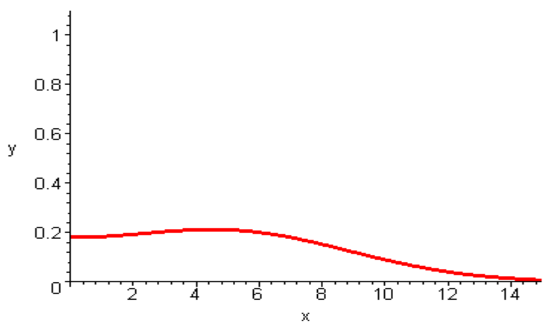


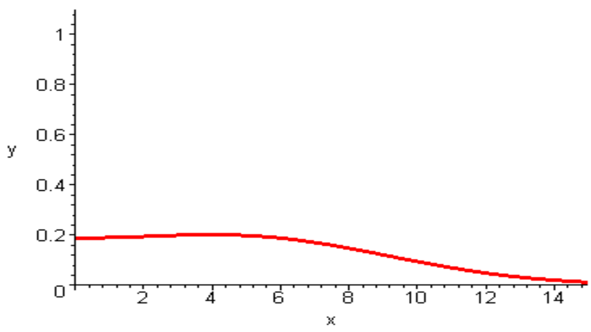


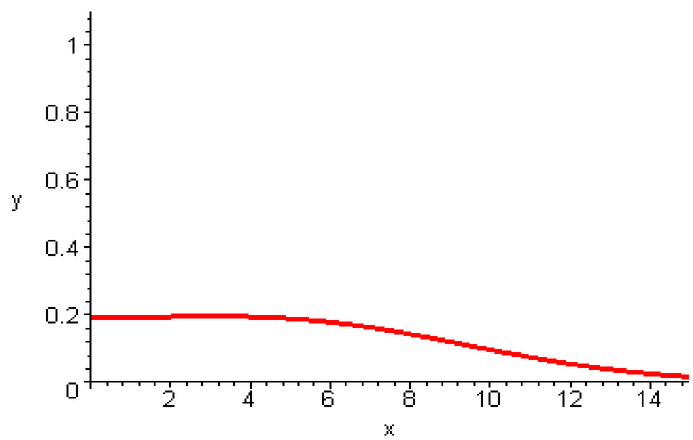












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