

## INDIVIDUAL RISK SOME ISSUES ABOUT THE MODEL

Jumakulov X. Q.

Kokand State Pedagogical Institute,  
Candidate of Sciences in Physics and Mathematics, Associate Professor

Makhmudova N. A.

Master Kokand State Pedagogical Institute

### ANNOTATION

In this article, the analysis of financial risks, which is the main part of insurance activity, the concept of studying the individual risk model of insurance and some examples are presented.

**Keywords:** Individual risk model, insurance, insurance events, Central Limit Theorem, composition of distributions.

The individual risk model is one of the simplest models designed to calculate the probability of an insurance company's accident, and it requires the fulfillment of the following conditions for insurance cases:

1. Insurance cases are analyzed for a relatively short period of time (usually 1 year). Therefore, inflation and investment income are not taken into account.
2. The number of contracts (insured persons)  $N$  is fixed and not random.
3. Insurance premiums are entered before the contract is concluded and there is no cash flow during the contract.
4. Each insurance contract is observed separately and with it a random variable  $X$  representing the associated insurance payment term is learned.

Defines individual risks in the insurance model

$$X_1, \dots, X_N$$

accidental amounts are considered unrelated (in particular, insurance payments are not made under several contracts at the same time).

Possible catastrophic events that occur in the individual risk model

$$S = X_1 + \dots + X_N$$

sum total of losses in the insurance portfolio is determined. If these total payments are  $u$  greater than the company's assets, the company will not be able to continue its activities according to the contracts, that is, it will go bankrupt. Therefore, the company's probability of bankruptcy

$$R = P(X_1 + \dots + X_N > u)$$

is defined by the equation In other words,  $S$  random probability can be considered as a distribution that can be considered additive to a random variable (ie  $R = 1 - P(X_1 + \dots + X_N \leq u)$ ). Therefore, since the amount of cumulative payments is a sum  $S$  of independent random variables, its distribution can be studied by classical methods of probability theory. One of the main methods is composition of distributions. Recall that if  $\eta_1$  and  $\eta_2$  are two independent nonnegative random variables, each of which has  $F_1(x)$  distribution functions  $\eta_1 + \eta_2$  and, respectively, then the  $F_2(x)$  distribution function of the sum

$$F(x) = F_1(x) * F_2(x) = \int_0^x F_1(x-y) dF_2(y)$$

formula . Apply this formula multiple times to an arbitrary number of random variables

$$F_S(x) = F_1 * F_2 * \dots * F_N = \int_0^x F_{N-1}^S(x-y) dF_N(y)$$

the formula .

If  $\eta_1$  and  $\eta_2$  random variables have distributions of the absolute continuous type and have density functions of , respectively ,  $p_1(x)$  and the  $p_2(x)$  density function of the  $\eta_1 + \eta_2$  sum of

$$p(x) = p_1 * p_2 = p_2 * p_1 = \int_0^x p_1(x-y) p_2(y) dy$$

formula . If the random variables  $\eta_1$  and  $\eta_2$  of discrete type (especially if they take non-negative values)

$$P_1(n) = P(\eta_1 = n), P_2(n) = P(\eta_2 = n)$$

distribution if has  $\eta_1 + \eta_2$  distributions

$$p(n) = P(\eta_1 + \eta_2 = n)$$

for

$$p(n) = \sum_{k=0}^n P_1(k) P_2(n-k) = P_1 * P_2$$

formula will be appropriate.

The method of composition faces fundamental difficulties in calculating the distributions of sums of random variables. Therefore, in the process of applying the distributions of total insurance payments to concrete calculations, the methods of approximating them with simple and accurate distributions come to the fore. In this case, it is important to approximate the sum distribution at large values of  $S = S_N$  with a normal (Gaussian) distribution based on the Central Limit Theorem .  $N$  This conclusion can be expressed in the following short expression: if  $X_1, \dots, X_N$  are uncorrelated and have a general uniform distribution, for which the second-order moment

$$\int x^2 dP(X_1 < x) < \infty$$

if available

$$S_N^* = \frac{X_1 + \dots + X_N - Na}{\sigma\sqrt{N}}, a = EX_1, \sigma^2 = DX_1$$

for the distribution of random variables

$$\sup_x |P(S_N^* < x) - \Phi(x)| \rightarrow 0, N \rightarrow \infty$$

limit relation is appropriate. Here

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du .$$

many different generalized versions of the Central Limit Theorem (for example, for various distributed independent and weakly dependent random variables). From a practical point of view, this is a theorem

$$P(S_N < x) \approx \Phi\left(\frac{x - ES_N}{\sqrt{\text{Var}S_N}}\right)$$

approximation is reasonable (  $N$  for large values of ). Standard normal distribution function in probability theory  $\Phi(x)$  and the corresponding density function

$$\Phi'(x) = p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

great depth, and a number of precise numerical tables of these functions have been compiled for their practical use.

**Problem 1.** An insurance company sold 300 fire insurance policies. The overview of the insurance portfolio is presented in the table below:

Number of contracts	Sum insured in the contract	Probability of an insurance event for one contract
100	400	0.05
200	300	0.06

The following are known:

- 1) for each contract, when an insurance event occurs, the amount of loss to be seen is evenly distributed between 0 and the sum insured;
- 2) probability of occurrence of more than one insurance event in the contract is 0;
- 3) insurance events occur independently of each other.

of the total premiums for the entire insurance portfolio .

**Solving.**  $k$  - let's define the indicator of the insurance event in the third contract  $I_k$ , if we define it as  $k$  the amount of the loss when the insurance event occurs, then  $Y_k$  it will be possible to write the insurance payment under the  $k$ -th contract . In  $X_k = I_k \cdot Y_k$  addition  $N = 300$ , the number of  $q_k = P(I_k = 1)$ ,  $M_k$  contracts  $k$  determines the total insurance payment for the contract. In this case, the total amount of payments for the insurance portfolio

$$S = X_1 + \dots + X_n$$

formula . So,

$$\text{Var}S = \text{Var}X_1 + \dots + \text{Var}X_n .$$

Since the contracts in the given 2 groups are not statistically different from each other

$$\text{Var}S = N^I \text{Var}X^I + \dots + N^{II} \text{Var}X^{II}$$

the equality holds and where  $N^I = 100$ ,  $N^{II} = 200$  are the number of contracts of the first and second type, respectively,  $\text{Var}X^I$ ,  $\text{Var}X^{II}$  and are the variances of the payoffs of the first and second type. A random amount representing the insurance premium

$$X = I \cdot Y$$

form , in which  $X$  and  $Y$  - the random variables are independent,  $I$  - the insurance event indicator (  $P(I = 1) = q$  ),  $Y - (0; M)$  will be a random variable uniformly distributed in the interval. By performing direct calculations

$$\text{Var}X = \frac{qM^2}{12} (4 - 3q)$$

equation .

From the last relationship

$$\text{Var}X^I = 2567,$$

$$\text{Var}X^{II} = 1719$$

relationship arises and hence

$$\text{Var}S = 100 \cdot 2567 + 200 \cdot 1719 = 650000.$$

**Problem 2.** Let's assume that the company  $N = 3000$  has registered life insurance. In this case, the probability of the client dying within a year is 0.3%. If the customer dies within a year, the company  $b = 250000$  pays soums, if not, nothing.

How much insurance premiums should the company collect (collect) to ensure that the probability of failure is 5% (0.05).

**Solving.** Usually, aggregate insurance payments are taken as a unit of monetary measure. In this case,  $i$  according to the  $i$ -th contract  $X_i$ -the insurance premium takes two values 0 and 1, respectively, with  $1-q$  probabilities  $q$ . That's why

$$EX_i = (1-q) \cdot 0 + q \cdot 1 = q = 0,003,$$

$$EX_i^2 = (1-q) \cdot 0 + q \cdot 1^2 = q,$$

$$\text{Var}X_i = EX_i^2 - (EX_i)^2 = q - q^2 \approx 0,003.$$

Now the total insurance payment

$$S = X_1 + \dots + X_n$$

for

$$ES = NEX_i = 3000 \cdot 0,003 = 9,$$

$$\text{Var}S = N\text{Var}X_i \approx 3000 \cdot 0,003 = 9$$

values.

sum of the  $S$  centralized and normalized insurance premium, we get the following result for the company's default probability:

$$P(S \leq u) = P\left(\frac{S - ES}{\sqrt{\text{Var}S}} \leq \frac{u - ES}{\sqrt{\text{Var}S}}\right) \approx P\left(\frac{S - ES}{\sqrt{\text{Var}S}} \leq \frac{u - 9}{3}\right) \approx \Phi\left(\frac{u - 9}{3}\right).$$

If we assign the probability of failure to be 5%, the  $\frac{u - 9}{3}$  amount

$$x_{95\%} = x_{95} = 1,645$$

must be equal to the  $\Phi(x)$  number (these were taken from existing tables for the normal distribution).

So in absolute numbers

$$u = [3 \cdot 1,645 + 9] \cdot ES = 13,935 \cdot ES = 3483750 \text{ soums}$$

should be.

**Problem 3.** Density function of the total payment amount for medical insurance

$$p(x) = \frac{1}{1000} e^{-x/1000}, x > 0.$$

the contract, a premium of 100 soums more than the payment is set for this amount.

If 100 contracts are concluded, find the approximate probability that the loss of the insurance company will be greater than the accumulated premium.

(  $P \approx 15,87\%$  ) option.

**Instructions for solving the problem . If we determine** the insurance premium for  $X$  one contract with a random quantity, its density function

$$p(x) = \lambda e^{-\lambda x}, x > 0, \lambda = \frac{1}{1000}.$$

With direct calculations

$$EX = \int_0^{\infty} x e^{-\lambda x} dx = 1000,$$

$$EX^2 = 2 \int_0^{\infty} x^2 e^{-\lambda x} dx = \frac{2}{\lambda^2} = 2000000,$$

$$\text{Var}X = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2} = 1000000$$

we make sure it is.

So one contract premium

$$p = EX + 100 = 1100,$$

i.e. all premiums collected

$$P = N \cdot p = 110000.$$

Insurance payment for all contracts

$$S = X_1 + \dots + X_N$$

is , where  $N = 100$  (number of contracts),  $X_i$  -  $i$  is the amount of insurance payment under the  $i$ -th contract (random) and its distribution function

$$P(X_i < x) = F(x) = 1 - e^{-\lambda x}, x > 0.$$

Now we are interested

$$P(S > Np) = P\left(\frac{S - ES}{\sqrt{\text{Var}S}} > \frac{Np - ES}{\sqrt{\text{Var}S}}\right)$$

applying the Central Limit Theorem for  $P \approx 15,87\%$  probability .

## Literature

1. Rotar V.I. Actuarial models. The Mathematics and insurance. Chapman&Hall/CRC-London, New York, 2007, 627 pp.
2. Bowers N. L., Gerber H. U., Hickman J. C., Jones D. A., Nesbitt C. J. Actuarial Mathematics. Itasca, Illinois: The Society of Actuaries, 1986, 644 pp.
3. Фалин Г. И. Математический анализ рисков в страховании. Российский юридический издательский дом, Москва, 1994, 130 стр.
4. Фалин Г. И., Фалин А. И. Теория риска для актуариев в задачах, М.: Изд-во «Мир», 2004, 239 стр.
5. Formanov Sh.Q. Ehtimolliklar nazariyasi. Darslik. Toshkent, "Universitet", 2014, 268 bet.
6. Sh.Q.Formanov. Aktuar matematika. Universitetlar, Iqtisodiyot oliy o'quv yurtlari talabalari va magistrarlari uchun darslik. Toshkent, "Universitet", 2018.
7. Юсупова, Д. Ш., & Исабаев, М. М. (2022). ОТНОШЕНИЕ УЧИТЕЛЕЙ К ИНКЛЮЗИВНОМУ ОБРАЗОВАНИЮ В КАЗАХСТАНЕ: КЕЙС ОБЩЕОБРАЗОВАТЕЛЬНЫХ ШКОЛ ГОРОДА АЛМАТЫ. Central Asian Economic Review, (5), 76-89.

8. Исабаева, М. М. (2014). ПРОБЛЕМЫ ПРОФОРИЕНТАЦИОННОЙ РАБОТЫ В СОВРЕМЕННОЙ ШКОЛЕ. In Научный потенциал молодежи в решении задач модернизации России (pp. 345-346).
9. Исабаева, М. М. (2012). Наркомания–общий враг человечества. Молодой ученый, (2), 265-267.
10. Сейтказиева, А. М., Исабаев, М. М., & Раушанов, Е. М. (2019). ПОВЫШЕНИЕ КОНКУРЕНТОСПОСОБНОСТИ ФИРМЫ В РАМКАХ ИНДУСТРИАЛЬНОЙ ПОЛИТИКИ: ЛИТЕРАТУРНЫЙ ОБЗОР. Economics: the strategy and practice, 14(4), 43-52.
11. Usmonova, M., & Mo'Minova, M. (2022). O'QUVCHILARNING BIOLOGIYA FANIDAN KREATIV FIKRLASH QOBILIYATINI RIVOJLANTIRISHDA XALQARO PISA DASTURINING ANAMIYATI. Science and innovation, 1(B7), 1254-1257.
12. Turdaliev, A., Usmonova, M., & Matholiqov, R. (2022). ОЛИЙ ТАЪЛИМ ТИЗИМИДА ЎҚИТУВЧИНИНГ МЕТОДИК КОМПЕТЕНТЛИГИНИ МОЎЯТИ. Science and innovation, 1(B6), 450-455.
13. Turdaliev, A., Usmonova, M., & Matholiqov, R. (2022). THE ESSENCE OF THE TEACHER'S METHODOLOGICAL COMPETENCE IN THE HIGHER EDUCATION SYSTEM. Science and Innovation, 1(6), 450-455.
14. Usmonova, M. (2022). SPECIFITY OF INTERACTIVE METHODS IN LANGUAGE LESSONS. Science and innovation, 1(B5), 165-168.
15. Mahmudovna, A. M., & Isaboeva, M. M. (2022). Forms of organizing the cognitive activity of students in the process of solving problems and exercises in biology. Web of Scientist: International Scientific Research Journal, 3(7), 68-76.
16. Mahmudovna, A. M., & Isaboeva, M. M. (2022). Forms of organizing the cognitive activity of students in the process of solving problems and exercises in biology. Web of Scientist: International Scientific Research Journal, 3(7), 68-76.
17. Шамотова, О. Ш. (2022, July). ВЛИЯНИЕ МОТИВАЦИИ НА СТУДЕНТОВ В ПРОЦЕССЕ УРОКА. In INTERNATIONAL CONFERENCE: PROBLEMS AND SCIENTIFIC SOLUTIONS. (Vol. 1, No. 2, pp. 277-280).
18. SHUKURDINOVNA, S. O., & KIZI, K. D. I. Pedagogical Problems of Creating English Textbooks. JournalNX, 7(1), 109-112.
19. Sh, S. O., & Kazakbayeva, D. I. Pedgogical problems of creating English textbooks. Journal NX, 7(1).
20. Sh, Shamatova O., and D. I. Kazakbayeva. "Pedgogical problems of creating English textbooks." Journal NX 7.1.
21. Tukhtasinova, D. T. (2022, September). HOW TO TEACH ENGLISH LANGUAGE MEDICAL ENGINEERING SPECIALTY STUDENTS. In INTERNATIONAL SCIENTIFIC CONFERENCE" INNOVATIVE TRENDS IN SCIENCE, PRACTICE AND EDUCATION" (Vol. 1, No. 2, pp. 157-162).
22. Nozimjon O'g'li, S. S. (2022). CAUSES OF THE ORIGIN OF OSTEOCHONDROSIS, SYMPTOMS, DIAGNOSIS AND TREATMENT METHODS. Conferencea, 76-77
23. Юсупова, М. Н., & Ахмедова, М. М. (2020). МЕВАЛИ ДАРАХТЛАРНИ ЗАРАРКУНАНДАЛАРИГА УЙЁУНЛАШГАН КУРАШ ЧОРАЛАРИ. ЖУРНАЛ АГРО ПРОЦЕССИНГ, 2(8).
24. ТУРДИЕВА, О. М., ТОЖИБОЕВА, С. Х., & ТУРСУНОВА, Ш. А. (2015). О ПРЕДОТВРАЩЕНИИ УСТАЛОСТИ У ШКОЛЬНИКОВ. In БУДУЩЕЕ НАУКИ-2015 (pp. 422-426).
25. Tursunova, S. A., & Mamasoliev, S. T. ALGOFLORA OF TYPICAL GRAY SOILS FOR CONTINUOUS TILLAGE. Chief Editor.

26. Рузиматов, Р. Я., Махкамов, Г. М., Отажонова, С. Р., & Турсунова, Ш. А. (2017). Промышленное развитие в Коканде, причины экологических проблем (1956-1975гг.). Высшая школа, (6), 77-78.
27. Тошматова, Ш. Р. (2016). Показатели достоверности и нарушения подразделений экологических ниш тлей. Молодой ученый, (20), 50-53.
28. Toshmatova, S. R., & Usmonov, S. O. (2021). Biological aspects of human adaptation to environmental conditions. ACADEMICIA: An International Multidisciplinary Research Journal, 11(3), 2185-2188.
29. Kalonova, M., Tashmatova, R. V., & Mukhamadiev, N. K. (2020). Preparation of melanin from silkworm wastes and studying its physical and chemical characteristics. CENTRAL ASIAN JOURNAL OF MEDICAL AND NATURAL SCIENCES, 1(2), 8-12.
30. Muminova, R. N., & Tashmatova, R. S. (2021). Bioecological features and significance of higher aquatic plants of the syr darya basin. ASIAN JOURNAL OF MULTIDIMENSIONAL RESEARCH, 10(4), 939-943.
31. Toshmatova, S. R., & Ernazarov, I. (2021). THE IMPORTANCE OF THE PROBLEM OF BIOREMEDIATION AS AN IMPORTANT SCIENTIFIC AND PRACTICAL PROBLEM IN THE FIELD OF HUMAN ACTIVITY. Экономика и социум, (1-1), 274-276.
32. ТОШМАТОВА, Ш. Р., ЭРНАЗАРОВ, З. М., & ИБРАГИМОВА, Д. А. ILMIY XABARNOMA. НАУЧНЫЙ ВЕСТНИК. ILMIY XABARNOMA. НАУЧНЫЙ ВЕСТНИК Учредители: Андижанский государственный университет им. ЗМ Бабура, (4), 48-55.
33. Toshmatova, S. R., Ernazarov, Z. M., & Ibragimova, D. A. RESULTS OF ANALYSIS OF AN APPLE OF RED BLOOD APHID (ERIOSOMA LANIGERIUM) IN THE RESEARCH AREA. ILMIY XABARNOMA, 54.