NOVATEUR PUBLICATIONS
JournalNX- A Multidisciplinary Peer Reviewed Journal
ISSN No: 2581 - 4230
VOLUME 7, ISSUE 5, May. -2021

ANALYTICAL STUDY ON THE SINGLE AND MULTIVARIABLE CALCULUS AND MULTIPLE INTEGRATION

Dr P K Dewedi ,
Professor, Research Supervisor, SunRise University, Alwar

Shivanand Saini,
Research Scholar, Sun Rise University, Alwar

ABSTRACT
This course places a strong focus on solving mathematical and narrative issues. One requires a lot of experience solving issues in order to become an expert problem solver. You will get better at solving issues as you encounter more of them because patterns will begin to appear in both the problems and the solutions that work. If you set aside some time each day to solve difficulties, you will learn the quickest and most effectively.

1. INTRODUCTION

We often the (x, y) coordinate system requires that positive x values be written right of the point of departure and positive y values be written above the origin. To rephrase, we take "rightward" to mean "positive x-direction" and "upward" to mean "positive y-direction" unless otherwise stated. The x- and y-axes typically have the same scale in purely mathematical contexts. The angle formed by the x-axis and the line from the origin to the point (a, a), for instance, is 45 degrees (not to mention the y-axis). In many cases, letters besides x and y are used in applications., and alternative scales are frequently used for the horizontal and vertical axes. Consider dropping anything from a window and seeing how the object's height changes over the course of a second. It makes sense to use the letter t to represent time (the duration of the item after release) and the letter h to represent height. You have a height h that corresponds to each t (let's say at one-second intervals). The (t, h) coordinate plane may be used to depict this data once it has been tabulated. The first quadrant is the "northeast" part of the plot, where both coordinates are positive; the second, third, and fourth quadrants are counted off anticlockwise, thus the northwest, southwest, and southeast are the second, third, and fourth quadrants, respectively.

1.2 Lines

When two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are present, only one line may be drawn through both of them. The ratio of y to x is referred to as the slope of this line. Slope is often indicated m: $m = \Delta y/\Delta x = \frac{(y_2-y_1)}{(x_2-x_1)}$. The shortest path between two places, for instance, (1, −2) and (3, 5) has slope (5 + 2)/(3 − 1) = 7/2.

EXAMPLE 1.1 The tax rate for a single parent was 15% on the first $26050 of income, as per the U.S. federal income tax schedules from 1990. In addition, 28% of the amount between $26050 and $67200 and 33% of the amount above $67200 had to be paid if the taxable income fell between $26050 and $134930. (if any). Use mathematical language to make sense of the information about the tax brackets

(15%, 28%, 33%) and to plot the tax amount in offset of the taxable income of a graph (y-axis). When translated to decimal form, the percentages 0.15, 0.28, and 0.33 indicate straight line gradients that make up the tax graph for the corresponding tax bands. For tax purposes, a polygonal graph is usedbecause it is constructed from numerous straight lines at varying angles. The first line climbs from the origin (0,0) with a slope of 0.15 until it passes through the point x = 26050. (i.e., it rises 15 for every increase of 100 in the x-direction). The graph "bends upward" when the slope changes to 0.28. If you move position in the horizontal plane from x = 26050 to x = 67200, you'll see that the line climbs by 28 for every 100 steps. At x = 67200, the line's 0.33-degree slope continues in an upward direction.

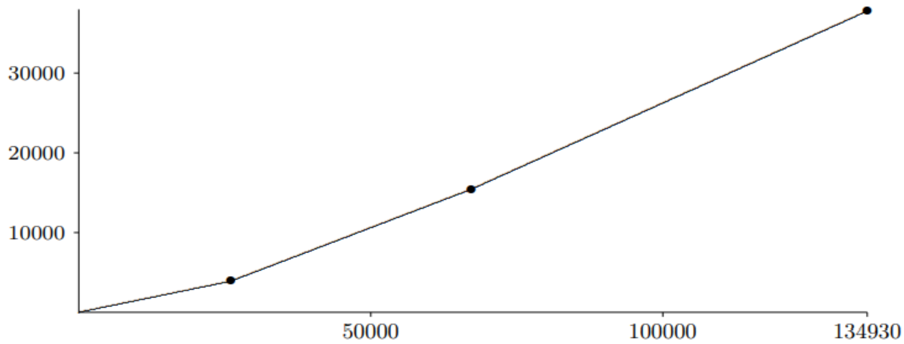

Figure 1.1 Tax vs. income

The equation for a straight line is most often written as y = mx+b. In this case, m is the line's slope, and the equation says that if you raise x by 1, you must also increase y by m. Y grows by y = mx if x is increased by x. Since this is where the line crosses the y-axis, the value b is known as the y-intercept. With the help of the formula, we can determine the slope between two given points on a line m = $\frac{(y_2-y_1)}{(x_2-x_1)}$. The y-intercept may be determined after a point and its slope are known by putting either of the points' coordinates in the equation: $y_1 = mx_1 + b$, or $b = y_1 + mx_1$. As an alternative, one may use the "point-slope" version with respect to the straight line equation, which is obtained by multiplying ($y\ y_1$) by ($x\ x_1$) to get ($y\ y_1$) = m($x\ x_1$), the point-slope form. It is possible to alter this further to get y = mx $mx_1 + y_1$, which is effectively the "mx + b" form.

1.2.1Calculating the Relative Size of Two Circles

Remember that two points (x_1, y_1) and (x_2, y_2) are separated from one another by a horizontal distance of x and a vertical distance of y. (In actuality, "distance" often refers to "space in the good direction. The context makes it clear that both X and Y are signed distances. An actual (positive) distance between two locations calculates the hypotenuse of a right triangle given its base and height |Δx| and |Δy|, as shown in figure 1.2. The distance between them is then calculated using the Pythagorean theorem, which says that the value is proportional to the square root of the product of the squares of the horizontal and vertical distances

$$\text{distance} = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The interval between two points, for instance,A(2, 1) and B(3, 3) is $\sqrt{(3-2)^2 + (3-1)^2} = \sqrt{5}.$

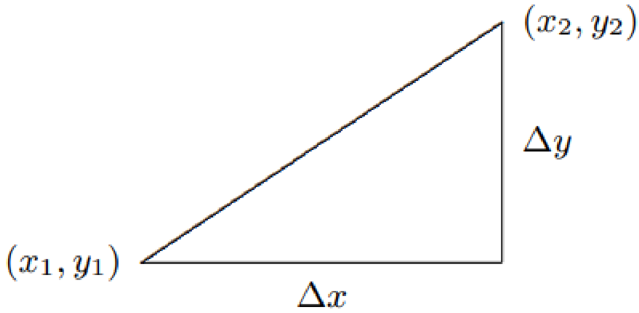

Figure 1.2 Negative Δx and positive Δy distance between two places.

Consider the situation where we need to calculate the distance from a location (x, y) to the origin as a specific instance of the distance formula. The distance formula states, this is $\sqrt{(x-0)^2+(y-0)^2} = \sqrt{x^2+y^2}$.

Only if and only if, can we say that the coordinates (x, y) are r units away from the zero point $\sqrt{x^2+y^2} = r$ or if we equalise everything: x2 + y2 = r2. The equation for a circle with centre and radius r is as follows. As a concrete illustration, the unit circle r = 1, and its equation is x2 + y2 = 1.

A distance r from a point C to a point (x, y) is the case if and only ifC(h, k) is any fixed point if $\sqrt{(x-h)^2+(y-k)^2} = r$,i.e., if and only if

$$(x-h)^2+(y-k)^2 = r^2.$$

The circle of radius r and centering at this point has the following equation (h, k). For instance, the formula for a circle having a centre and a radius of 5 (0, 6) (x − 0)2 + (y − −6)2 = 25, or x 2+(y+6)2 = 25. If we expand this, we get x_2+y_2+12y+36 = 25 or x_2+y_1+12y+11 = 0, However, the first form is frequently the most helpful.

2. MULTIPLE INTEGRATION, SECONDARY

2.1Sufficient Space and Typical Height

Consider the surface f(x, y), which you may first consider to reflect actual topography—perhaps a hilly area. What is the average surface height (or average elevation of the terrain) in a given area? We begin by considering how we may approximating the solution, as we do with the majority of such issues. Assume the area has the following dimensions: [a, b] [c, d]. As shown in figure 1.3, we may split grid with m columns, n rows, and a rectangle as its base. In each subdivision, we choose x values x0, x1,..., xm1, and in the y direction, we do the same. We calculate the height of the surface at each of the locations (xi, yj) into a smaller area on the grid: f (xi , yj). Now, depending on how small the grid is, the average of these heights should be quite near to the surface's average height:

$$\frac{f(x_0,y_0)+f(x_1,y_0)+\cdots+f(x_0,y_1)+f(x_1,y_1)+\cdots+f(x_{m-1},y_{n-1})}{mn}$$

We anticipate that this approximation will converge to a constant number, the real average height of the surface, as both m and n approach infinity. This does occur for moderately decent functions.

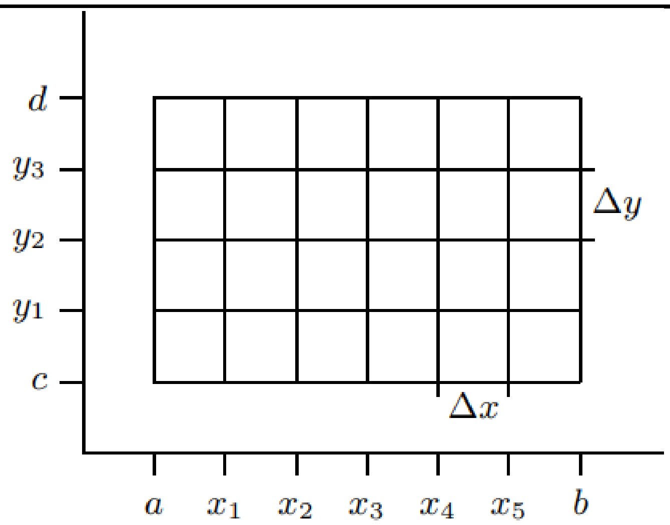

Figure 1.3It's a square of [a, b] × [c, d].

We may express the calculation as follows in sigma notation:

$$\frac{1}{mn}\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} f(x_j, y_i) = \frac{1}{(b-a)(d-c)}\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} f(x_j, y_i)\frac{b-a}{m}\frac{d-c}{n}$$

$$= \frac{1}{(b-a)(d-c)}\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} f(x_j, y_i)\Delta x\Delta y.$$

The two components of this item have practical meaning: The area of the rectangle is obviously given by (b a)(d c), twice the total multiplies the surface height at a location by the area of one of the smaller rectangles we've cut the larger one into, yielding mn terms of the form f(xj, yi)xy. Figure 1.4 shows that the space occupied by a long, narrow, and tall box is represented by the sum of the expressions f(xj, yi) ΔxΔy for each of the small rectangles. The volume above and below the rectangle may be approximatively represented by the formula R = [a, b] x [c, d]. The double total becomes the actual volume below the surface when we take the limit as m and n approach infinity, and we divide this by the surface area to get the volume below the surface (b - a)(d - c) to get the mean height. The double sum in this example is, in a sense, the most important, since it is used in so many contexts; dividing by (b - a) (d - c) is a trivial extra step that permits the calculation of an average. We provide the maximum value of such a double sum is represented by a unique symbol, just as we did in the case of a single variable:

$$\lim_{m,n\to\infty}\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} f(x_j, y_i)\Delta x\Delta y = \iint_R f(x,y)\,dx\,dy = \iint_R f(x,y)\,dA,$$

the R-regional There are two f's in the double integral. Instead of writing dx and dy, the shorter and more "generic" notation is dA.since it denotes a tiny amount of area without defining an order for the variables x and y. In this notation, the surface's typical height is

$$\frac{1}{(b-a)(d-c)}\iint_R f(x,y)\,dA.$$

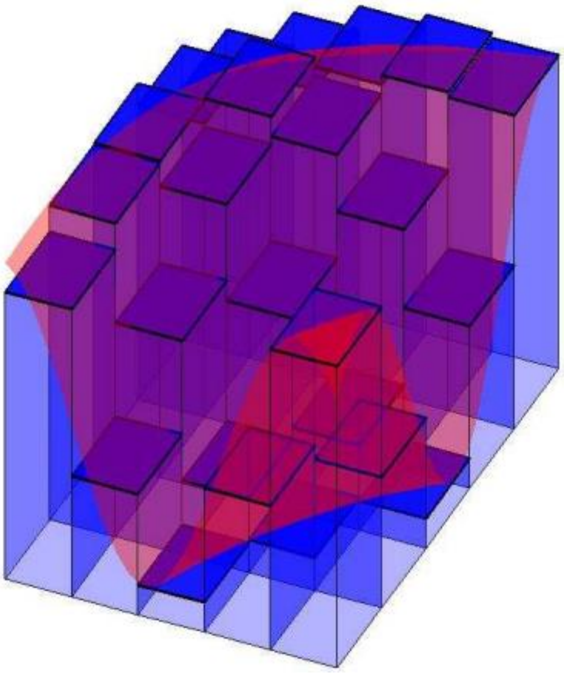

Figure 1.4Calculating the volume roughly hidden behind a flat surface

Of course, the next concern is how to calculate such double integrals. While a Basic Calculus Theorem in Two Dimensions would seem necessary at first glance, it really only requires a single variable. Rewriting the double sum will highlight the desired order in which the terms will be added:

$$\sum_{i=0}^{n-1}\left(\sum_{j=0}^{m-1} f(x_j, y_i)\Delta x\right)\Delta y.$$

Only the value of xj is changing in the total enclosed in parentheses; yi is momentarily constant. This sum has the appropriate form to become an integral as m increases to infinity:

$$\lim_{m\to\infty}\sum_{j=0}^{m-1} f(x_j, y_i)\Delta x = \int_a^b f(x, y_i)\,dx$$

Therefore, the total is Taking the limit as m becomes infinitely large.

$$\sum_{i=0}^{n-1}\left(\int_a^b f(x, y_i)\,dx\right)\Delta y$$

Naturally, this integral has distinct values for different values of yi; That is to say, it is in fact a function that is applied toyi:

$$G(y) = \int_a^b f(x, y)\,dx$$

Adding back in the substitution results in the total.

$$\sum_{i=0}^{n-1} G(y_i)\Delta y$$

This sum may be interpreted in a good way. The cross-sectional area of the area under the surface f(x, y), or when y = yi, is represented by the value G(yi). It is possible to think of the amount A solid with area G(yi) and thickness Δy has volume G(yi)y. The crusty upper part of a loaf of bread is a surface similar to f(x, y). G(yi)y is the volume of a single bread slice, taking into account its thickness and cross-sectional area. Total bread volume may be approximately calculated by adding these. With the exception of the requirement that the cross-sections be somehow "the same" there, this method is remarkably similar to the one we used to calculate volumes. This "sliced loaf" approximation is shown in Figure 1.5 using the same surface. Interestingly, this sum seems to be exactly the kind of sum that becomes an integral, thus

$$\lim_{n\to\infty}\sum_{i=0}^{n-1} G(y_i)\Delta y = \int_c^d G(y)\,dy$$

$$= \int_c^d \int_a^b f(x,y)\,dx\,dy$$

This indicates that we will calculate the inner integral first, with y assumed to be fixed for the time being. Please explain this to me in simple terms. First, we'll calculate the anti-derivative of x, and then we'll do the standard thing and make the substitutions x = a and x=b before we take the derivative away. The final result will be an expression that uses y but no x. If the outer integral has y as its variable, then the issue is a simple one-variable one.

EXAMPLE 1.2 Figure 1.5 shows the function sin(xy)+6/5 on [0.5, 3.5]×[0.5, 2.5]. This area's subsurface volume is

$$\int_{0.5}^{2.5}\int_{0.5}^{3.5}\left\{\sin(xy)+\frac{6}{5}\right\}dx\,dy$$

The inner integral is

$$\int_{0.5}^{3.5}\left\{\sin(xy)+\frac{6}{5}\right\}dx = \frac{-\cos(xy)}{y}+\frac{6x}{5}\Big|_{0.5}^{3.5} = \frac{-\cos(3.5y)}{y}+\frac{\cos(0.5y)}{y}+\frac{18}{5}$$

Sadly, this yields a function for which we are unable to identify a straightforward anti-derivative. We could use Sage or a comparable piece of software to estimate the integral and finish the task.

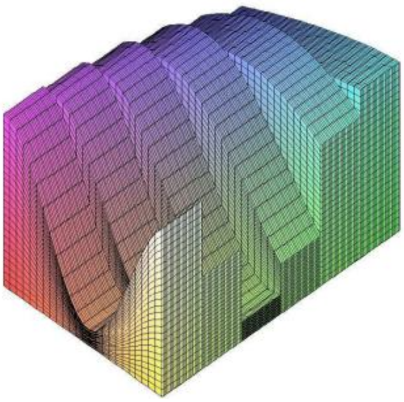

Figure 1.5Slice-based volume approximation of a solid surface

This yields a volume of around 8.84, indicating that the average height is about 8.84/6 ≈ 1.47.

Due relating to the characteristics of addition and multiplication, namely their commutativity and associativity, the original double total may be rewritten as:

$$\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} f(x_j, y_i)\Delta x\Delta y = \sum_{j=0}^{m-1}\sum_{i=0}^{n-1} f(x_j, y_i)\Delta y\Delta x.$$

If we continue the progression from above, the inner sum becomes an integral:

$$\lim_{n\to\infty}\sum_{i=0}^{n-1} f(x_j, y_i)\Delta y = \int_c^d f(x_j, y)\, dy,$$

after which the outer total becomes an integral:

$$\lim_{m\to\infty}\sum_{j=0}^{m-1}\left(\int_c^d f(x_j, y)\, dy\right)\Delta x = \int_a^b \int_c^d f(x, y)\, dy\, dx$$

For example, we might calculate the integrals with regard to x first, then y, or vice versa. This is equivalent to make a second cut in the bread loaf perpendicular to the first. We have not yet shown Fubini's Theorem, which asserts that, if the function is good enough, In whatever order of integration, a double integral is always equal to the sum of its two constituent single integrals.

EXAMPLE 1.3 We compute $\iint_R 1 + (x-1)^2 + 4y^2\, dA,$ where R = [0, 3] × [0, 2], in two ways.

First,

$$\int_0^3 \int_0^2 \{1 + (x-1)^2 + 4y^2\}\, dy\, dx = \int_0^3 \left\{y + (x-1)^2 y + \frac{4}{3}y^2\right\}\Big|_0^2\, dx$$

$$= \int_0^3 \left\{2 + 2(x-1)^2 y + \frac{32}{3}\right\} dx$$

$$= \int_0^3 2 + 2(x-1)^2 + \frac{32}{3}\, dx$$

$$= 2x + \frac{2}{3}(x-1)^3 + \frac{32}{3}x\Big|_0^3$$

$$= 6 + \frac{2}{3}\cdot 8 + \frac{32}{3}\cdot 3 - \left(0 - 1\cdot\frac{2}{3} + 0\right)$$

= 44.

In the other order:

$$\int_0^2 \int_0^3 \{1 + (x-1)^2 + 4y^2\}\, dx\, dy = \int_0^2 \left\{x + \frac{(x-1)^2}{3} + 4y^2 x\,\Big|_0^3\right\}\, dy$$

$$= \int_0^2 \left\{3 + \frac{8}{3} + 12y^2 x + \frac{1}{3}\right\}\, dy$$

$$= 3y + \frac{8}{3}y + 4y^3 + \frac{1}{3}y\Big|_0^2$$

$$= 6 + \frac{16}{3} + 32 + \frac{2}{3}$$

= 44.

3. CONCLUSION

One requires a lot of experience solving issues in order to become an expert problem solver. You will get better at solving issues as you encounter more of them because patterns will begin to appear in both the problems and the solutions that work. If you set aside some time to solve difficulties each day, you will learn the quickest and best.

REFERENCES

1. V.L. Chaurasia and K. Tak, Double integral relations involving a general class of polynomials and the multivariable H-function, Bull. Calcutta. Math. Soc, 93(2) (2001), 115-120.
2. J. Prathima V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journalof Engineering Mathematics (2014) , 2014, 1-12.
3. A.K. Rathie, A new generalization of generalized hypergeometric functions, Le Matematiche Fasc. II, 52 (1997), 297-310.
4. H.M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), 183-191.
5. H.M.Srivastava and R.Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976)., 167-197.
6. H.M. Srivastava and N.P. Singh, The integration of certain products of the multivariable H-function with a general class of polynomials. Rend. Circ. Mat. Palermo. Vol 32 (No 2) (1983), 157-187.
7. C. Szego, (1975), Orthogonal polynomials. Amer. Math. Soc. Colloq. Publ. 23 fourth edition. Amer. Math. Soc. Providence. Rhodes Island, 1975.
8. D. L. Suthar, R. K. Parmar, S. D. Purohit, Fractional calculus with complex order and generalized hypergeometric functions, Nonlinear Sci. Lett. A, 8 (2) (2017), 156–161.
9. S. D. Purohit, S. L. Kalla, D. L. Suthar, Fractional integral operators and the multiindexMittag-Leffler functions, Sci. Ser. A. Math. Sci., 21 (2011), 87–96.
10. D. Kumar, S. D. Purohit, J. Choi, Generalized fractional integrals involving product of multivariable H-function and a general class of polynomials, J. Nonlinear Sci. Appl., 9 (2016), 8–21.