
STUDY OF NUCLEAR STRUCTURE OF ^{45,47}Ti ISOTOPES BY USING OXBASH CODE

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Abstract

Titanium ^{47,45}₂₂Ti has 26 isotopes from ³⁸Ti to ⁶³Ti five of these isotopes stable like ⁴⁶Ti, ⁴⁷Ti, ⁴⁸Ti, ⁴⁹Ti, ⁵⁰Ti . titanium isotopes ⁴⁵Ti, ⁴⁷Ti have neutrons (N=23,25) with 5 to 7 nucleons outside of closed shell , shell model is utilized to determine the energy levels and B (E2) with use of the shell model code OXBASH for windows and the effective interactions F7MBZ and F742, with the spin-parity of the valence nucleons taken into account. The calculated energy levels and B(E2) values mostly agreed with the available experimental data.

Keywords: Energy levels, Shell model, OXBASH code, B (E2).

INTRODUCTION

The nuclear shell model is just one of many that have been created to explain the nucleus [1], as well as certain additional characteristics like the spins, parities, and B(E2). It provides the theoretical foundation for a microscopic description of nuclear properties that is primarily dependent on the utilization of effective interactions [2], There are a number of "standard" effective interactions, including the USD and Cohen-Kurath interactions for the p and SD shells [3]. in order to determine the nuclear structural characteristics of both ground and excitations according to the nuclear shell concept. Those states' wave functions are required, to obtain these wave functions, the shell-model program OXBASH is used. Titanium's nucleus structure was calculated using the Windows program OXBASH. with using the *f7* with (F7MBZ & F742) effective interactions [4] . Similarities exist between this model and the atomic shell model of electrons. Similar to how valence electrons outside of a closed shell can characterize atomic behavior and qualities, the value nucleons (protons or neutrons) situated in close-proximity shells with magic numbers (2,8,20,28,50,82, and 126) dictate the nuclear attributes of a nucleus. Highly stable nuclei with magic numbers have unique properties. Atomic behavior and attributes can be defined by the valence electrons that exist in an open shell. Highly stable nuclei with magic numbers have unique properties [5]. This work's goal is to investigate the decreased transition probabilities and level schemes for ⁴⁵Ti, ⁴⁷Ti, and odd isotopes use the most recent OXBASH for Windows version the energy levels of some ⁴⁵Ti states and ⁴⁷Ti Compared to the most recent data, the figures calculated in this paper show [6].

2. Theory

Traditional calculations using the shell model, Typically, one would determine the energy levels using related to a closed shell as opposed to the system's overall energy, as well as for a solitary nucleon outside the doubly miraculous core. In this instance, it is assumed that energy is an eigenvalue of the Hamiltonian H0. when the whole Hamiltonian is represented as and there are many nucleons beyond the core [7].

$$H = \sum_{k=1} H_o + \sum_{k \leq l} V_{kl} \quad \dots \dots (1)$$

Where $\sum_{k \leq l} V_{kl}$ is residual two-body interactions [8], Generally, a quantum mechanics solution to the Schrödinger steps to an equation have been essential [9]. so that Schrödinger equation can be expressed in writing.

$$H = |\psi_n \rangle = E_n |\psi_n \rangle \quad \dots \dots (2)$$

Where

$$H = H_o + H_1 \quad \dots \dots (3)$$

$$H_o = \sum_{i=1}^A (T_o + U_i) \quad \dots \dots (4)$$

$$H_1 = \sum_{i < j}^A (V_{ij}^{NN} - \sum_{i=1}^A U_i) \quad \dots \dots (5)$$

To disentangle [10] the nuclear Hamiltonian one-body potentials have been proposed as the integral of one-body terms. Which characterizes the nucleons' free-moving nature plus the interaction H_1 [11]. The Schrödinger equation can be solved using the mean field potential to obtain the single-particle wave functions in the extreme single-particle shell model, and the resulting quantum numbers (n, l, j) for the energy levels of the particles. The quantum number (n) denotes the quantity of nodes in the radial wave function. The orbital angular momentum is denoted by the symbol l . Where j is the overall angular momentum brought about by the coupling of the intrinsic nucleon spin = $\frac{1}{2}$. The two options to the orbital angular momentum $j = \pm 1/2$ [12].

3. Calculations and Discussion

The calculations were performed in the nuclear shell model f7 with the windows program OXBASH. An m-scheme Slater determinant Basis is used in the code. A projection approach is used to create wave functions with good angular momentum J and isospin T [13]. Shell model calculation of $^{45,47}Ti$ isotopes were carried out for the space model $(1F7/2)$, for the aforementioned isotopes, with neutrons $(N=23$ and $25)$ above the near ^{40}Ca core In addition to 5 nucleons outside core for ^{45}Ti and 7 nucleons outside core for ^{47}Ti [14], and effective interactions F7MBZ, F742.

3.1 Levels of energy

The purpose of this investigation is to Determine the nuclei that are in close proximity to ^{45}Ti because of the significant role that these nuclei play in recent developments in astrophysical applications. the determined energy levels and presented low-lying state experimental results for odd-odd nuclei. However, On the left, you can see the results of our calculations and right-hand experimental info for any band [9].

3.1.1 Energy levels of ^{45}Ti

For ^{45}Ti isotope using (F7MBZ) interactions is shown in the table1. When measured against the experimental data that is presented, through our theoretical calculations, the energy value of the ground level was obtained as 0, and its angular momentum was $(5/2^-)$ while the momentum was $(7/2^-)$ in the available practical results, but in our theoretical calculations we obtained a value of (0.274) MeV with the same angular momentum $(7/2^-)$. A good agreement was obtained for the values of the practical energies $(1.35349, 1.46824, 3.01537, 4.3449)$ MeV corresponding to the angular momentum $(9/2_1, 11/2_1, 15/2_1, 19/2_1)$, when compared with the calculated theoretical values, The total momentum and parity of the unconfirmed practical energies $(1.5217, 2.0147, 2.500, 4.723, 5.1800, 10.1535)$ MeV

was confirmed for the angular momentum ($3/2_{-1}$, $3/2_{-2}$, $7/2_{-2}$, $7/2_{-4}$, $3/2_{13-}$, $25/2_{-1}$) are confirmed when compared with the calculated theoretical values , And The experimental energy value(2.5314 ,4.8552 ,5.2399, 7.3420) MeV was confirmed for angular momentum ($5/2_{+}$, $17/2_{+}$, $17/2_{+}$, $23/2_{+}$) with negative parity In our calculations, We found the values of energies for a specific angular momentum close to the values of the practical energies (3.200 ,5.540 , 6.0067 ,7.8307 ,9.6435) that have no specific angular momentum, and thus we expect that its angular momentum is the theoretically calculated momentum ($9/2_{2-}$, $15/2_{2-}$, $15/2_{-3}$, $15/2_{-6}$, $11/2_{-9}$). Through our calculations, we noticed that there are sixty-two levels with total angular momentum and parity that were not matched by any available practical value.

Table 1. Excitation energy predicted by (F7MBZ) interactions and observed excitation energies for the ^{45}Ti nucleus are compared.

Theoretical values for F7MBZ		Experimental values		Theoretical values for F7MBZ		Experimental values	
J^π	E (MeV)	E (MeV)	J^π	J^π	E (MeV)	E (MeV)	J^π
$5/2_{1^-}$	0	0.000	$7/2_{1^-}$	$23/2_{1^-}$	6.912	7.3420	($23/2_{1^+}$)
$7/2_{1^-}$	0.274	-----	-----	$9/2_{6^-}$	6.973	-----	-----
$9/2_{1^-}$	1.705	1.35359	$9/2_{1^-}$	$15/2_{4^-}$	7.06	-----	-----
$11/2_{1^-}$	1.763	1.46824	$11/2_{1^-}$	$17/2_{3^-}$	7.289	-----	-----
$3/2_{1^-}$	1.966	1.5217	$3/2_{1^-}$ to $9/2_{1^-}$	$11/2_{-6}$	7.333	-----	-----
$3/2_{2^-}$	2.658	2.0147	$3/2_{2^-}$ to $9/2_{2^-}$	$17/2_{4^-}$	7.343	-----	-----
$7/2_{2^-}$	2.769	2.500	$5/2_{1^-}$, $7/2_{1^-}$	$5/2_{6^-}$	7.395	-----	-----
$5/2_{2^-}$	3.223	2.5314	$1/2_{1^-}$, $3/2_{1^-}$, $5/2_{1^+}$ (+)	$9/2_{7^-}$	7.450	-----	-----
$15/2_{1^-}$	3.561	3.01537	$15/2_{1^-}$	$13/2_{6^-}$	7.549	-----	-----
$9/2_{2^-}$	3.748	3.200	-----	$11/2_{7^-}$	7.662	-----	-----
$7/2_{3^-}$	3.871	-----	-----	$15/2_{5^-}$	7.664	-----	-----
$13/2_{1^-}$	3.943	-----	-----	$19/2_{3^-}$	7.726	-----	-----
$1/2_{1^-}$	3.957	-----	-----	$15/2_{6^-}$	7.892	7.8307	-----
$17/2_{1^-}$	3.982	4.8552	($17/2_{+}$)	$13/2_{7^-}$	7.918	-----	-----
$9/2_{3^-}$	4.385	-----	-----	$11/2_{8^-}$	8.135	-----	-----
$11/2_{2^-}$	4.426	-----	-----	$21/2_{2^-}$	8.162	-----	-----
$5/2_{3^-}$	4.612	-----	-----	$27/2_{1^-}$	8.232	-----	-----
$19/2_{1^-}$	4.628	4.3449	$19/2_{-}$	$7/2_{8^-}$	8.401	-----	-----
$13/2_{2^-}$	4.716	-----	-----	$9/2_{8^-}$	8.576	-----	-----
$7/2_{4^-}$	4.745	4.723	($7/2_{-}$)	$17/2_{-5}$	8.600	-----	-----
$11/2_{3^-}$	4.909	-----	-----	$9/2_{9^-}$	8.783	-----	-----

9/2 ₄ ⁻	5.483	-----	-----	13/2 ₈ ⁻	8.826	-----	-----
15/2 ₂ ⁻	5.532	5.540	-----	23/2 ₂ ⁻	8.828	-----	-----
11/2 ₄ ⁻	5.549	-----	-----	19/2 ₄ ⁻	8.837	-----	-----
3/2 ₃ ⁻	5.636	5.180	1/2 ⁻ , 3/2 ⁻	15/2 ₇ ⁻	8.962	-----	-----
13/2 ₃ ⁻	5.837	-----	-----	17/2 ₆ ⁻	9.124	-----	-----
17/2 ₂ ⁻	5.925	5.2399	(17/2 ⁺)	5/2 ₇ ⁻	9.216	-----	-----
9/2 ₅ ⁻	5.951	-----	-----	1/2 ₃ ⁻	9.238	-----	-----
15/2 ₃ ⁻	6.000	6.0067	-----	3/2 ₅ ⁻	9.242	-----	-----
11/2 ₅ ⁻	6.037	-----	-----	25/2 ₁ ⁻	9.276	10.1535	(25/2 ₁ ⁻)
5/2 ₄ ⁻	6.074	-----	-----	11/2 ₉ ⁻	9.644	9.6435	-----
7/2 ₅ ⁻	6.159	-----	-----	3/2 ₆ ⁻	9.786	-----	-----
5/2 ₅ ⁻	6.313	-----	-----	7/2 ₉ ⁻	9.944	-----	-----
13/2 ₄ ⁻	6.399	-----	-----	15/2 ₈ ⁻	10.069	-----	-----
19/2 ₂ ⁻	6.433	-----	-----	13/2 ₉ ⁻	10.116	-----	-----
7/2 ₆ ⁻	6.676	-----	-----	21/2 ₃ ⁻	10.169	-----	-----
3/2 ₄ ⁻	6.722	-----	-----	9/2 ₁₀ ⁻	10.245	-----	-----
1/2 ₂ ⁻	6.764	-----	-----	5/2 ₈ ⁻	10.413	-----	-----
21/2 ₁ ⁻	6.814	-----	-----	19/2 ₅ ⁻	10.806	-----	-----
13/2 ₅ ⁻	6.882	-----	-----	7/2 ₁₀ ⁻	10.985	-----	-----
7/2 ₇ ⁻	6.899	-----	-----	5/2 ₂ ⁻	11.147	-----	-----

For ⁴⁵Ti isotope using (F742) interactions is shown in the table2. When measured against the experimental data that is presented, through our theoretical calculations, the energy value of the ground level was obtained as 0, and its angular momentum was ((5/2-) while the momentum was (7/2-) in the available practical results, but in our theoretical calculations we obtained a value of (0.093) MeV with the same angular momentum (7/2-). A good agreement was obtained for the values of the practical energies (0.03653, 1.35349, 1.46824 , 3.01537 ,4.3449 ,6.1630 ,7.1434) MeV corresponding to the angular momentum (3/2₋₁ , 9/2₋₁, 11/2₋₁, 15/2₋₁, 19/2₋₁ , 23/2₋₁ , 27/2₋₁) when compared with the calculated theoretical values , The total momentum and parity of the unconfirmed practical energies (2.0147,2.500 , 4.723 , 5.180) MeV was confirmed for the angular momentum (3/2₋₂ , 7/2₋₂, 7/2₋₅, 1/2₋₂)are confirmed when compared with the calculated theoretical values , And The experimental energy value(2.5314, 2.8494 ,2.9329, 3.9376 ,5.030 ,6.7579, 7.3420 ,8.2892) MeV was confirmed for angular momentum (5/2₊₂, 1/2₊₁ , 13/2₊₁, 11/2₊₂ ,5/2₊₅, 21/2₊₂ , 23/2₊₂, 25/2₊₂) with negative parity In our calculations, The total angular momentum has been determined for the values of the practical energies for which parity (2.8494 , 3.9376) has not been determined corresponding to the angular momentum (1/2₁, 11/2₂) when compared with the available practical values, We found the values of energies for a specific angular momentum close to the values of the practical energies (3.156, 3.200 ,5.540 , 6.0067 ,7.8307 ,9.6435) that have no specific angular momentum, and thus we expect that its angular momentum is the theoretically calculated momentum (9/2₂⁻, 7/2₂⁻, 3/2₄⁻ , 5/2₆⁻, 7/2₉⁻ , 19/2₅⁻). Through our calculations, we noticed that there are fifty-five levels with total angular momentum and parity that were not matched by any available practical value.

Table 2. Excitation energy predicted by (F742) interactions and observed excitation energies for the ^{45}Ti nucleus are compared.

		Experimental values		Theoretical values for F742		Experimental values	
J^π	E (MeV)	E (MeV)	J^π	J^π	E (MeV)	E (MeV)	J^π
5/2 ₁	0	0.000	7/2 ₁ ⁻	5/2 ₆	6.010	6.0067	(23/2 ₁ ⁺)
7/2 ₁	0.093	-----	-----	15/2 ₄	6.137	-----	-----
3/2 ₁	1.526	0.03653	3/2 ₁ ⁻	9/2 ₇	6.137	-----	-----
9/2 ₁	1.529	1.35349	9/2 ₁ ⁻	11/2 ₆	6.253	-----	-----
11/2 ₁	1.571	1.46824	11/2 ₁ ⁻	21/2 ₁	6.338	-----	-----
3/2 ₂	2.231	2.0147	3/2 ₁ ⁻ to 9/2 ₁ ⁻	13/2 ₆	6.415	-----	-----
7/2 ₂	2.365	2.500	5/2 ₁ ⁻ , 7/2 ₁ ⁻	11/2 ₇	6.432	-----	-----
5/2 ₂	2.772	2.5314	1/2 ⁻ , 3/2 ⁻ , 5/2 ⁻ (+)	17/2 ₃	6.478	-----	-----
9/2 ₂	3.136	3.1560	-----	17/2 ₄	6.505	-----	-----
7/2 ₃	3.229	3.200	-----	23/2 ₁	6.522	6.1630	23/2 ⁻
15/2 ₁	3.236	3.01537	15/2 ⁻	13/2 ₇	6.635	-----	-----
1/2 ₁	3.290	2.8494	1/2 ⁻ , 3/2 ⁻ , 5/2 ⁻ (+)	15/2 ₅	6.644	-----	-----
13/2 ₁	3.490	2.9329	(13/2 ⁺)	11/2 ₈	6.73	-----	-----
11/2 ₂	3.594	3.9376	(11/2 to 15/2)	15/2 ₆	6.778	-----	-----
9/2 ₃	3.626	-----	-----	7/2 ₈	6.842	-----	-----
17/2 ₁	3.720	-----	-----	19/2 ₃	6.927	-----	-----
5/2 ₃	3.752	-----	-----	9/2 ₈	7.054	-----	-----
7/2 ₄	3.998	-----	-----	9/2 ₉	7.173	-----	-----
13/2 ₂	4.113	-----	-----	17/2 ₅	7.309	-----	-----
11/2 ₃	4.194	-----	-----	1/2 ₃	7.329	-----	-----
19/2 ₁	4.222	4.3449	-----	3/2 ₅	7.341	-----	-----
15/2 ₄	4.551	-----	-----	5/2 ₇	7.393	-----	-----
3/2 ₃	4.657	-----	-----	21/2 ₂	7.403	6.7579	(21/2 ⁺)
11/2 ₄	4.684	-----	-----	13/2 ₈	7.425	-----	-----
9/2 ₄	4.694	-----	-----	15/2 ₇	7.526	-----	-----
5/2 ₄	4.844	-----	-----	19/2 ₄	7.695	-----	-----
9/2 ₅	4.916	-----	-----	3/2 ₆	7.809	-----	-----
7/2 ₅	5.096	4.723	(7/2 ₁) ⁻	11/2 ₉	7.826	-----	-----
5/2 ₅	5.111	5.030	(3/2 ⁺ , 5/2 ⁺)	17/2 ₆	7.857	-----	-----
13/2 ₃	5.148	-----	-----	27/2 ₁	7.874	7.1434	27/2 ₁ ⁻
11/2 ₅	5.223	-----	-----	7/2 ₉	7.933	7.8307	-----

1/2 ₂	5.252	5.180	1/2 ⁻ , 3/2 ⁻	23/2 ₂	8.037	7.3420	(23/2 ⁺)
15/2 ₃	5.269	-----	-----	9/2 ₁₀	8.185	-----	-----
17/2 ₂	5.316	-----	-----	5/2 ₈	8.259	-----	-----
13/2 ₄	5.366	-----	-----	13/2 ₉	8.282	-----	-----
7/2 ₆	5.389	-----	-----	15/2 ₈	8.380	-----	-----
3/2 ₄	5.521	5.540	-----	25/2 ₁	8.453	8.2892	(25/2 ⁺)
7/2 ₇	5.742	-----	-----	7/2 ₁₀	8.707	-----	-----
9/2 ₆	5.846	-----	-----	21/2 ₃	8.845	-----	-----
19/2 ₂	5.861	-----	-----	11/2 ₁₀	8.992	10.985	-----
13/2 ₅	5.862	-----	-----	19/2 ₂	9.153	9.6435	-----

3.1.2 Energy levels of ⁴⁷Ti

For ⁴⁷Ti isotope using (F7MBZ) interactions is shown in the table3. When measured against the experimental data that is presented, through our theoretical calculations, the energy value of the ground level was obtained as 0, and its angular momentum was (7/2⁻) while the momentum was (5/2⁻) in the available practical results, but in our theoretical calculations we obtained a value of (0.002) MeV with the same angular momentum (5/2⁻). A good agreement was obtained for the values of the practical energies (1.54965, 1.44425, 1.79380 ,2.1632 ,2.6194, 2.74887,2.5482 , 3.99394 , 4.49411, 3.8271, 4.67290 , 8.0051) MeV corresponding to the angular momentum (3/2⁻₁, 11/2⁻₁, 1/2⁻₁, 3/2⁻₂, 7/2⁻₂, 15/2⁻₂, 3/2⁻₃,15/2⁻₂, 19/2⁻₁, 7/2⁻₅, 17/2⁻₂, 27/2⁻₁) when compared with the calculated theoretical values , The total momentum and parity of the unconfirmed practical energies (2.4062, 2.7576, 2.6823, 2.8095 , 3.7271 ,3.780 , 4.095, 5.433, 6.067, 6.3664) MeV was confirmed for the angular momentum (9/2⁻₂, 7/2⁻₃,11/2⁻₃, 5/2⁻₃, 13/2⁻₂, 9/2⁻₄,1/2⁻₂, 3/2⁻₅ , 1/2⁻₃, 21/2⁻₁) are confirmed when compared with the calculated theoretical values , The total angular momentum has been determined for the values of the practical energies for which parity (3.7018) has not been determined corresponding to the angular momentum (7/2₄) when compared with the available practical values. , We found the values of energies for a specific angular momentum close to the values of the practical energies (1.67 ,2.695 , 2.8002 ,3.724 , 4.04 ,3.961 ,4.112 ,4.243 ,4.264 ,4.303 ,4.518 , 4.541 ,4.553, 4.708 ,4.898 ,5.102 ,5.125 ,5.195 ,5.301 , 5.372 ,5.451 ,5.478, 5.635 , 5.702 ,5.774 , 6.095 ,6.129 ,6.195 ,6.209 ,6.234 ,6.265 , 6.304, 6.364 ,6.402 ,6.449, 6.474, 6.514 ,6.554 , 6.624 ,6.645 ,6.823, 6.854 ,6.882, 6.903 ,6.917 ,6.002 ,7.038 ,7.123 ,7.205 ,7.225 ,7.4806) that have no specific angular momentum, and thus we expect that its angular momentum is the theoretically calculated momentum (5/2₂⁻, 11/2₂⁻, 9/2₃⁻ , 17/2₁⁻, 5/2₄⁻ , 15/2₃⁻ , 11/2₄⁻ , 9/2₅⁻ , 3/2₄⁻ , 13/2₃⁻, 13/2₄⁻ , 11/2₅⁻, 5/2₅⁻, 15/2₄⁻ , 9/2₆⁻ , 13/2₅⁻ , 19/2₂⁻ , 11/2₆⁻ , 9/2₇⁻ , 5/2₆⁻, 7/2₆⁻ , 7/2₇⁻ , 9/2₈⁻ , 15/2₅⁻ , 11/2₇⁻, 13/2₆⁻, 19/2₃⁻, 7/2₈⁻ , 5/2₇⁻, 17/2₃⁻,23/2₁⁻ , 11/2₆⁻ , 15/2₆⁻ , 9/2₉⁻ , 7/2₉⁻ , 11/2₉⁻, 3/2₆⁻ , 15/2₇⁻ , 13/2₇⁻ , 9/2₁₀⁻ , 5/2₈⁻ , 15/2₈⁻ , 17/2₄⁻ , 13/2₈⁻ , 5/2₉⁻ , 11/2₁₀⁻ , 19/2₄⁻ , 17/2₅⁻ , 13/2₉⁻, 5/2₁₀⁻ , 23/2₃⁻). Through our calculations, we noticed that there are nineteen levels with total angular momentum and parity that were not matched by any available practical value.

Table 3. Excitation energy predicted by (F7MBZ) interactions and observed excitation energies for the ⁴⁷Ti nucleus are compared.

Theoretical values for F7MBZ		Experimental values		Theoretical values for F7MBZ		Experimental values	
J^{π}	E (MeV)	E (MeV)	J^{π}	J^{π}	E (MeV)	E (MeV)	J^{π}
7/2 ₁	0	0	5/2 ⁻	3/2 ₅	5.798	5.433	1/2 ⁻ , 3/2 ⁻
5/2 ₁	0.002	-----	-----	21/2 ₁	5.833	5.19744	21/2 ⁻
3/2 ₁	0.801	1.54965	3/2 ⁻	1/2 ₃	5.903	6.067	(1/2 ⁻ , 3/2 ⁻)
11/2 ₁	1.213	1.44425	11/2 ⁻	13/2 ₆	6.006	6.095	-----
1/2 ₁	1.447	1.79380	11/2 ₁ ⁻	19/2 ₃	6.058	6.129	-----
5/2 ₂	1.548	1.670	-----	7/2 ₈	6.106	6.195	-----
9/2 ₁	1.582	-----	-----	5/2 ₇	6.261	6.209	-----
9/2 ₂	2.131	2.4062	(9/2 ⁻)	17/2 ₃	6.277	6.234	-----
3/2 ₂	2.41	2.1632	3/2 ⁻	23/2 ₁	6.285	6.265	-----
7/2 ₂	2.495	2.6194	7/2 ⁻	11/2 ₈	6.319	6.304	-----
15/2 ₁	2.577	2.74887	15/2 ⁻	15/2 ₆	6.388	6.364	-----
11/2 ₂	2.604	2.6950	-----	9/2 ₉	6.442	6.402	-----
9/2 ₃	2.805	2.8002	-----	7/2 ₉	6.458	6.449	-----
7/2 ₃	2.872	2.7576	7/2 ⁻ to 13/2 ⁻	11/2 ₉	6.459	6.474	-----
3/2 ₃	2.98	2.9800	3/2 ⁻	3/2 ₆	6.55	6.514	-----
11/2 ₃	3.172	2.68230	11/2(-)	15/2 ₇	6.774	6.554	-----
5/2 ₃	3.209	2.8095	5/2 ⁻ , 7/2 ⁻ , 9/2 ⁻	13/2 ₇	6.835	6.624	-----
13/2 ₁	3.365	-----	-----	9/2 ₁₀	6.844	6.645	-----
15/2 ₂	3.547	3.99394	15/2 ⁻	5/2 ₈	6.85	6.823	-----
13/2 ₂	3.577	3.7271	(13/2 ⁻)	15/2 ₈	6.869	6.854	-----
17/2 ₁	3.643	3.724	-----	17/2 ₄	6.886	6.882	-----
9/2 ₄	3.804	3.7800	3/2(-) to 9/2 ⁻	13/2 ₈	6.893	6.903	-----
5/2 ₄	4.021	4.040	-----	21/2 ₂	6.949	6.3664	(21/2 ⁻)
15/2 ₃	4.082	3.961	-----	5/2 ₉	6.967	6.9170	-----
7/2 ₄	4.129	3.7018	7/2, 9/2, 3/2, 5/2 ⁻	11/2 ₁₀	7.037	7.002	-----
11/2 ₄	4.138	4.112	-----	19/2 ₄	7.194	7.038	-----
9/2 ₅	4.242	4.243	-----	17/2 ₅	7.224	7.123	-----
3/2 ₄	4.279	4.264	-----	13/2 ₉	7.242	7.205	-----
1/2 ₁	4.29	4.095	1/2 ⁻ , 3/2 ⁻	5/2 ₁₀	7.293	7.225	-----
13/2 ₃	4.321	4.303	-----	17/2 ₆	7.488	-----	-----
19/2 ₁	4.336	4.49411	19/2 ⁻	3/2 ₇	7.501	-----	-----
13/2 ₄	4.410	4.518	1/2 ⁻ , 3/2 ⁻	21/2 ₃	7.582	-----	-----
7/2 ₅	4.538	3.8271	7/2 ⁻	7/2 ₁₀	7.637	-----	-----

11/2 ₅	4.595	4.541	----	15/2 ₉	7.659	----	----
5/2 ₅	4.61	4.553	----	19/2 ₅	7.833	----	----
15/2 ₄	4.967	4.708	----	23/2 ₂	7.867	----	----
9/2 ₆	4.984	4.898	----	23/2 ₃	8.101	7.4806	----
13/2 ₅	5.123	5.102	----	25/2 ₁	8.169	----	----
19/2 ₂	5.147	5.125	----	17/2 ₇	8.211	----	----
11/2 ₆	5.206	5.195	----	15/2 ₁₀	8.224	----	----
9/2 ₇	5.297	5.301	----	1/2 ₄	8.269	----	----
5/2 ₆	5.335	5.372	----	13/2 ₁₀	8.334	----	----
17/2 ₂	5.46	4.6729	----	19/2 ₆	8.523	----	----
7/2 ₆	5.462	5.451	----	27/2 ₁	8.703	8.0051	27/2 ₁
7/2 ₇	5.506	5.478	----	3/2 ₈	8.713	----	----
9/2 ₈	5.637	5.635	----	21/2 ₄	9.578	----	----
15/2 ₅	5.746	5.702	----	17/2 ₈	10.135	----	----
11/2 ₇	5.768	5.774	----	3/2 ₉	11.911	----	----

For ⁴⁷Ti isotope using (F742) interactions is shown in the table3. When measured against the experimental data that is presented, through our theoretical calculations, the energy value of the ground level was obtained as 0, and its angular momentum was (7/2-) while the momentum was (5/2-) in the available practical results, but in our theoretical calculations we obtained a value of (0.126) MeV with the same angular momentum (5/2-). A good agreement was obtained for the values of the practical energies (1.44425, 1.79380 ,1.25209 ,2.1632 , 2.74887, 2.5482 , 3.28773 ,3.99394 , 4.49411, 4.67290 ,5.19744 , 6.0886 , 8.0051) MeV corresponding to the angular momentum (11/2₋₁, 1/2₋₁, 9/2₋₁, 3/2₋₂, 15/2₋₁ , 3/2₋₃ ,13/2₋₁ , 15/2₋₂, 19/2₋₁ , 17/2₋₂, 21/2₋₁, 23/2₋₁ , 27/2₋₁) when compared with the calculated theoretical values , The total momentum and parity of the unconfirmed practical energies (1.2507 , 2.4062, 2.2971 ,2.5482 , 2.7576 , 2.8463 , 2.8095 , 3.780 , 3.4845, 5.433, 5.013, 5.540, 6.3664, 5.615 , 6.3664 , 6.333) MeV was confirmed for the angular momentum (3/2₋₁ , 9/2₋₂,7/2₋₂ , 3/2₋₃, 7/2₋₃ , 11/2₋₃,5/2₋₃ ,5/2₋₄ , 11/2₋₄,3/2₋₄ , 3/2₋₅ , 1/2₋₃ , 3/2₋₆ , 3/2₋₇ , 21/2₋₂,3/2₋₈) are confirmed when compared with the calculated theoretical values , The total angular momentum has been determined for the values of the practical energies for which parity (2.668 ,3.3689 ,3.4005) has not been determined corresponding to the angular momentum (9/2₃, 9/2₄, 13/2₂) when compared with the available practical values , We found the values of energies for a specific angular momentum close to the values of the practical energies (1.67 ,3.654 ,3.724 , 3.839 ,3.961 ,3.889 ,3.961 ,4.303 , 4.518, 4.541 ,4.553 ,4.605 ,4.670 ,4.708 ,4.811, 4.876 ,4.898 ,5.043 ,5.102 ,5.195 ,5.301 , 5.372 ,5.451, 5.478 ,5.635, 5.67 ,5.702 ,5.746 , 5.755 , 5.774 ,5.836 ,5.872 ,5.937 ,6.013 ,6.095 ,6.129 , 6.195 ,6.206 , 6.234 ,6.265 ,6.304 , 6.364, 6.387 ,6.402 ,6.449 , 6.514 ,6.787 ,7.018 ,7.038 ,7.095 , 7.123 ,7.141 ,7.205 ,7.225) that have no specific angular momentum, and thus we expect that its angular momentum is the theoretically calculated momentum (5/2₋₂, 17/2₋₁, 15/2₋₃, 9/2₋₅, 13/2₋₃ , 13/2₋₄ , 11/2₋₅ , 9/2₋₆ , 13/2₋₅ , 15/2₋₄ , 11/2₋₆ , 5/2₋₆ , 9/2₋₇ , 7/2₋₆ , 7/2₋₇ , 9/2₋₈ , 19/2₋₂ , 11/2₋₇ , 7/2₋₈ ,15/2₋₅ , 5/2₋₇ , 13/2₋₆ , 9/2₋₉ , 7/2₋₉ , 11/2₋₈ , 11/2₋₉ , 5/2₋₈ , 19/2₋₃ , 15/2₋₆ , 17/2₋₃,5/2₋₉ , 9/2₋₁₀ , 15/2₋₇ , 13/2₋₇ , 5/2₋₁₀ , 15/2₋₁₀ , 11/2₋₁₀ , 13/2₋₈ , 17/2₋₄ , 7/2₋₁₀ , 13/2₋₉ , 17/2₋₅ , 17/2₋₆ , 19/2₋₄ , 15/2₋₉ , 1/2₋₄ , 21/2₋₃ , 13/2₋₁₀ , 19/2₋₅ , 15/2₋₁₀ , 17/2₋₇ , 23/2₋

$2, 19/2^-, 23/2^-$). Through our calculations, we noticed that there are nine levels with total angular momentum and parity that were not matched by any available practical value.

Table 4. Excitation energy predicted by (F742) interactions and observed excitation energies for the ^{47}Ti nucleus are compared.

Theoretical values for F742		Experimental values		Theoretical values for F742		Experimental values	
J^π	E (MeV)	E (MeV)	J^π	J^π	E (MeV)	E (MeV)	J^π
$7/2_1$	0	0	$5/2^-$	$7/2_8$	5.143	5.102	$1/2^-, 3/2^-$
$5/2_1$	0.126	-----	-----	$17/2_2$	5.148	4.6729	$17/2^-$
$3/2_1$	0.826	1.2507	$(1/2^-, 3/2^-)$	$15/2_5$	5.223	5.195	$(1/2^-, 3/2^-)$
$11/2_1$	1.276	1.44425	$11/2^-$	$5/2_7$	5.374	5.301	-----
$5/2_2$	1.428	1.670	-----	$13/2_6$	5.391	5.372	-----
$1/2_1$	1.517	1.7938	$1/2^-$	$9/2_9$	5.415	5.451	-----
$9/2_1$	1.636	1.25209	$9/2^-$	$3/2_6$	5.427	5.540	$1/2^-, 3/2^-$
$3/2_2$	2	2.1632	$3/2^-$	$7/2_9$	5.479	5.478	-----
$9/2_2$	2.016	2.4062	$(9/2^-)$	$11/2_8$	5.48	5.635	-----
$7/2_2$	2.284	2.2971	$5/2^-, 7/2^-$	$11/2_9$	5.603	5.670	-----
$11/2_2$	2.349	2.6823	$11/2(-)$	$5/2_8$	5.607	5.702	-----
$15/2_1$	2.546	2.74887	$15/2^-$	$21/2_1$	5.687	5.19744	$21/2^-$
$3/2_3$	2.547	2.5482	$3/2^-$	$19/2_3$	5.753	5.746	-----
$9/2_3$	2.549	2.668	$9/2, 13/2$	$15/2_6$	5.758	5.755	-----
$7/2_3$	2.62	2.7576	$7/2^-$ to $13/2^-$	$17/2_3$	5.759	5.774	-----
$11/2_3$	2.921	2.8463	$5/2^-$ to $11/2^-$	$5/2_9$	5.766	5.836	-----
$5/2_3$	2.926	2.8095	$5/2^-, 7/2^-, 9/2^-$	$9/2_{10}$	5.892	5.872	-----
$13/2_1$	3.219	3.28773	$13/2^-$	$15/2_7$	5.933	5.937	-----
$9/2_4$	3.261	3.3689	$7/2^-, 9/2^-, 11/2^-$	$13/2_7$	6.019	6.013	-----
$15/2_2$	3.351	3.99394	$15/2^-$	$5/2_{10}$	6.031	6.095	-----
$13/2_2$	3.389	3.4005	$7/2^-$ to $13/2^-$	$15/2_8$	6.054	6.129	-----
$5/2_4$	3.405	3.780	$3/2(-)$ to $9/2^-$	$11/2_{10}$	6.076	6.195	-----
$17/2_1$	3.562	3.654	-----	$13/2_8$	6.136	6.209	-----
$1/2_2$	3.595	-----	-----	$23/2_1$	6.14	6.0886	$23/2^-$
$11/2_4$	3.646	3.7018	$7/2, 9/2, 3/2, 5/2^-$	$3/2_7$	6.234	5.615	$1/2^-, 3/2^-$
$15/2_3$	3.713	3.724	-----	$17/2_4$	6.237	6.234	-----
$3/2_4$	3.714	3.4845	$(3/2^-)$	$7/2_{10}$	6.289	6.265	-----
$9/2_5$	3.738	3.839	-----	$13/2_9$	6.296	6.304	-----
$7/2_4$	3.738	-----	-----	$17/2_5$	6.452	6.364	-----
$5/2_5$	3.943	-----	-----	$21/2_2$	6.554	6.3664	$(21/2^-)$

13/2 ₃	3.969	3.961	-----	17/2 ₆	6.64	6.397	-----
7/2 ₅	3.982	-----	-----	19/2 ₄	6.667	6.402	-----
13/2 ₄	4.115	3.889	-----	15/2 ₉	6.769	6.449	-----
11/2 ₅	4.149	3.961	-----	1/2 ₄	6.819	6.514	-----
19/2 ₁	4.2	4.49411	19/2 ⁻	3/2 ₈	6.999	6.333	1/2 ⁻ , 3/2 ⁻
9/2 ₆	4.478	4.303	-----	21/2 ₃	7.011	6.787	-----
13/2 ₅	4.567	4.518	-----	13/2 ₁₀	7.088	7.018	-----
15/2 ₄	4.573	4.541	-----	19/2 ₅	7.133	7.038	-----
11/2 ₆	4.588	4.553	-----	17/2 ₁₀	7.19	7.095	-----
5/2 ₆	4.611	4.605	-----	17/2 ₇	7.275	7.123	-----
9/2 ₇	4.647	4.670	-----	23/2 ₂	7.326	7.141	-----
7/2 ₆	4.684	4.708	-----	19/2 ₆	7.596	7.205	-----
7/2 ₇	4.818	4.811	-----	23/2 ₃	7.622	7.225	-----
3/2 ₅	4.858	5.433	1/2 ⁻ , 3/2 ⁻	25/2 ₁	7.83	-----	-----
9/2 ₈	4.927	4.876	-----	27/2 ₁	8.386	8.0051	27/2 ⁻
19/2 ₂	4.929	4.898	-----	21/2 ₄	8.55	-----	-----
1/2 ₃	4.961	5.013	1/2 ⁻ , 3/2 ⁻	17/2 ₈	8.673	-----	-----
11/2 ₇	5.133	5.043	-----	3/2 ₉	9.461	-----	-----

3.2 B(E2) Calculations:

3.2.1 B(E2) for ⁴⁵Ti

Within the nuclear shell model, (F7MBZ & F742) projected that the chance of an electric quadruple transition B (E2) for ⁴⁵Ti would be lower. The transition probability was determined for each in-band transition assuming a pure E2 transition by using the harmonic oscillator potential (HO, b), Where b < 0. Core polarization effects have been taken into account by selecting the effective charges for the proton (ep=1.5e) and the neutron (en=1.5e). Table 5 ⁴⁵Ti was calculated with the help of the efficient interactions of F7MBZ and F742. Over all, there appears to be a fair amount of concordance between the computed results and the available experimental data.

Table 5. The B (E2) values for ⁴⁵Ti ground-state band. They use e²fm⁴ units, which match the experimental results.

J_i^-	→	J_f^-	Theory B(E2) (e ² fm ⁴)		Exp. B(E2) (e ² fm ⁴)
			F7MBZ	F742	
3/2	→	7/2	285.7	301.9	285.227
9/2	→	5/2	226.7	216.4	101.731
9/2	→	7/2	410	395.6	152.121
11/2	→	7/2	475.7	479.6	171.136
15/2	→	11/2	552.5	537.2	95.076
17/2	→	13/2	3.365	21.99	96.977

19/2	→	15/2	248.3	222.5	18.064
23/2	→	19/2	197.8	185.4	81.765
27/2	→	23/2	154.6	155.8	59.898

3.2.2 B(E2) for ^{47}Ti

Within the nuclear shell model, (F7MBZ & F742) projected that the chance of an electric quadruple transition B (E2) for ^{47}Ti would be lower. The transition probability was determined for each in-band transition assuming a pure E2 transition by using the harmonic oscillator potential (HO, b), Where $b < 0$. Core polarization effects have been taken into account by selecting the effective charges for the proton ($e_p=1.5e$) and the neutron ($e_n=0.816e$). Table 6 ^{47}Ti was calculated with the help of the efficient interactions of F7MBZ and F742. Over all, there appears to be a fair amount of concordance between the computed results and the available experimental data.

Table 6. the B (E2) values for ^{47}Ti ground-state band. They use $e^2\text{fm}^4$ units, which match the experimental results.

J_i^-	→	J_f^-	Theory B(E2) ($e^2\text{fm}^4$)		Exp. B(E2) ($e^2\text{fm}^4$)
			F7MBZ	F742	
7/2	→	5/2	244.2	240.4	244.758
9/2	→	7/2	66.64	87.21	186.016
9/2	→	5/2	221.6	197.7	68.532
11/2	→	9/2	78.83	80.56	393.710
11/2	→	7/2	311.1	302	166.435
3/2	→	7/2	277.4	258.8	38.182
3/2	→	5/2	254.5	239.8	3.231
1/2	→	3/2	119.8	107.5	9790.320
1/2	→	5/2	199.6	177.6	11.748
3/2 ₂	→	7/2	61.38	66.62	35.245
(9/2)	→	5/2	221.6	197.7	26.434
15/2	→	11/2	268.9	254.5	127.274
7/2 ₄	→	11/2	162.2	19.28	283.919
13/2	→	9/2	4.450	0.9131	2.154
(3/2)	→	7/2	277.4	258.8	45.035
17/2	→	15/2	63.88	67.58	587.419
7/2	→	11/2	466.6	453	420.984
15/2 ₂	→	11/2	22.26	25.78	5.874
19/2	→	17/2	54.96	62.47	16.644
19/2	→	15/2	149.7	117.5	27.413
17/2	→	13/2	109.6	115	195.806

21/2	→	17/2	53.18	51.1	489.516
23/2	→	21/2	49.90	47.05	196.785
23/2	→	19/2	113.7	93.73	43.077
27/2	→	23/2	66.13	66.13	186.016

4. CONCLUSIONS

All figures show, success in reaching an agreement nearly all energy levels of the isotopes of Titanium, and a proper reproduction of the level arrangement is made. We can evaluate practically any calculations in relation to ($F7MBZ$ & $F742$) data. Met with some success in replicating the level structure exhibited. Generally speaking, the greatest and most thorough results possess the biggest model space while performing calculations in the infinite sphere. In OXBASH, the model space is described based on the nucleon valence orbits that are now excited together with the outcomes of our calculations are often in agreement with experimental findings

7. References

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