NOVATEUR PUBLICATIONS
JournalNX- A Multidisciplinary Peer Reviewed Journal
ISSN No: 2581 - 4230
VOLUME 9, ISSUE 4, April. -2023

THE STUDY OF SINGLE AND DOUBLE INTEGRATION FORMULA

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Abstract

We introduce a new double transform called double Elzaki transform (modification of Smudu transform), where we will study this transform and their theorems on convergence. Also, we discuss the double new transform and it is convergent. Recently, Brychkov evaluated some new classes of definite and indefinite single and double integrals involving various elementary special functions and the logarithmic function. Single Integral is the building block of double and triple integrals, so if you are not able to do single integrals, you cannot do double and triple integrals. In calculus, and more generally in mathematical analysis, integration by parts or partial integration is a process that finds the integral of a product of functions in terms of the integral of the product of their derivative and antiderivative. In a very interesting, useful and popular research paper, Brychkov evaluated several integrals (single and double) of these types and gave several interesting special cases.

Keywords: Single, Double, Integration, Special Function, Transform.

INTRODUCTION

The subject of special functions, with the emergence of new problems in the field of applied sciences and engineering, is becoming increasingly rich and expanding. In the seventeenth century, Oxford Professor John Wallis presented the theory of the gamma function and elliptic integrals. Euler depicted the properties of the gamma function in 1730, and he also obtained the beta function integral in terms of the gamma function in 1972. Special functions are the product of an ancient branch of mathematics; their unified and rather complete theory dates back to the nineteenth century and continues to this day. That's why it is also known as golden age of the theory of special functions. In 1812, C. F. Gauss defined the hypergeometric series 2F1. In 1828 and 1836 Clausen and Kummer introduced 3F2 and 1F1 series respectively. Also, Appell defined this hypergeometric function for two variables and letter on Lauricella introduced them for several variables in 1893. However, the generalization of the 2F1 (Gaussian hypergeometric function) into the pFq (Generalized hypergeometric function), which diverges when $p > q + 1$, was a significant development in the theory of special functions. They have integral as well as series representations.

LITERATURE REVIEW

Ayant, Frédéric. (2020) In this paper some double finite integrals involving a general class og multivariable polynomials, multivariable A-function and multivariable Ifunction with general arguments have been evaluated. The result is believed to be new and is capable of giving a very large number of double or simpler integrals involving a large number of special functions and polynomials as its special cases. We shall see several corollaries at the end.

Ayant, Frédéric & Kumar, Dinesh. (2018) Recently Chaurasia and Gill, Chaurasia and Kumar have solved the one-dimensional integral equation of Fredholm type involving the product of special functions. We solve an integral equation involving the product of a class of multivariable polynomials, the multivariable H-function defined by Srivastava and Panda and the multivariable I-function defined by Prasad by the application of fractional calculus theory. The results obtained here are general in nature and capable of yielding a large number of results (known and new) scattered in the literature.

Dhali, Md & Hasan, Md & Selim, A & Barman, Nandita. (2020) Numerical integral is one of the mathematical branches that connect between analytical mathematics and computer. Numerical integration is a primary tool used by engineers and scientists to obtain an approximate result for definite integrals that cannot be solved analytically. Numerical double integration is widely used in calculating surface area, the intrinsic limitations of flat surfaces and finding the volume under the surface. A wide range of method is applied to solve numerical double integration for equal data space but the difficulty is arisen when the data values are not equal. In this paper we have tried to generate a mathematical formula of numerical double integration for unequal data spaces. Trapezoidal rule for unequal space is used to evaluate the formula. We also verified our proposed model by demonstrating some numerical examples and compared the numerical result with the analytical result.

Ayant, Frédéric. (2020) The object of this paper is to establish three finite double integrals involving certain product of special functions, the multivariables I-function and a general class of polynomials with general arguments. Our double integrals are quite general in character and a number of doubles integrals can be deduced as particular cases.

Mehdiyeva, Galina & Ibrahimov, Vagif & Imanova, Mehriban. (2019) It is known that in the construction of the numerical methods for solving of the initial-value problem of ODE in basically used the methods which have applied to the calculation of the definite integrals. Here for the computing of definite integrals propose to use the methods which have used in solving of the initial-value problem for the ODEs. The definite integrals express by the indefinite integrals which are the solutions of the above-mentioned problems. For the construction of more exact methods for calculation of the definite integrals here propose to use forward-jumping (advanced) methods and the hybrid methods. Here establishes some connection between the Gauss and hybrid methods. And also have determined some necessary conditions for which the coefficients of the proposed methods have to satisfy. Constructed stable methods with the degree $p \leq 8$. Shown that, how received here results can be applied to the computing of the double integrals. For this aim, determines some connection between double integrals and single definite integrals. By using this relation have constructed methods which are applied to calculate the double integrals. Advantages of this method illustrated by calculation of model double integral by the constructed here methods.

EVALUATION OF A DOUBLE INTEGRAL

In the theory of hypergeometric and generalized hypergeometric functions, classical summation theorems such as those of Gauss, Kummer and Bailey for the 2F1 series; Watson, Dixon, Whipple and Saalsch¨utz for the 3F2 series and others play an important role. It is well known that for every such summation theorems, we can evaluate a number of finite integrals involving hypergeometric or generalized hypergeometric functions and/or logarithmic functions. Furthermore, it is not out of place to mention here that the Gauss's hypergeometric function 2F1 and the confluent hypergeometric function 1F1 are the core of the special functions and almost all elementary functions can be obtained either as special cases or limiting cases of these functions. Thus, the finite integrals involving hypergeometric functions or the generalized hypergeometric function and/or the logarithmic function play an important role.

Preliminaries

In order to put our result in computable form, we need the following definitions.

Definition 1: The Psi function, denoted by Ψ(z), is defined as the logarithmic derivative of the gamma function Γ(z), that is,

$$\Psi(z)=\frac{\mathrm{d}}{\mathrm{d}z}\ln\Gamma(z)=\frac{\Gamma'(z)}{\Gamma(z)},\qquad\left(z\in\mathbb{C}\setminus\mathbb{Z}_0^-\right) \tag{1}$$

Definition 2: The Polygamma functions, denoted by Ψ(n) (z) (n ∈ N), are defined b

$$\Psi^{(n)}(z)=\frac{\mathrm{d}^{n+1}}{\mathrm{d}z^{n+1}}\ln\Gamma(z)=\frac{\mathrm{d}^n}{\mathrm{d}z^n}\Psi(z),\qquad\left(z\in\mathbb{C}\setminus\mathbb{Z}_0^-\right) \tag{2}$$

on, we present the main double integral formula. It is given in the form of a theorem.

Theorem 1: The following double integral formula holds true:

$$\int_0^1\int_0^1 x^{d-1}(1-x)^{c-2d}y^{d-1}(1-y)^{c-2d}(1-xy)^{-c}\ \ln^n\left(\frac{(1-x)(1-y)}{1-xy}\right)\mathrm{d}x\,\mathrm{d}y$$
$$=\frac{1}{2}\,\Gamma^2(d)\sum_{r=0}^{n-1}\binom{n-1}{r}\frac{\partial^r}{\partial c^r}A\cdot\frac{\partial^{n-r-1}}{\partial c^{n-r-1}}B \tag{3}$$

Where

$$A=\frac{\Gamma\left(\frac{1}{2}c\right)\Gamma\left(1+\frac{1}{2}c-2d\right)\Gamma(1+c-2d)}{\Gamma(c)\,\Gamma^2\left(1+\frac{1}{2}c-d\right)} \tag{4}$$

$$\ln A=\ln\Gamma\left(\frac{1}{2}c\right)-\ln\Gamma(c)+\ln\Gamma(1+c-2d)+\ln\Gamma\left(1+\frac{1}{2}c-2d\right)$$
$$-2\ln\Gamma\left(1+\frac{1}{2}c-d\right), \tag{5}$$

$$\frac{\partial}{\partial c}A=A\cdot B \tag{6}$$

$$B=\Psi(1+c-2d)-\Psi(c)+\frac{1}{2}\Psi\left(\frac{1}{2}c\right)+\frac{1}{2}\Psi\left(1+\frac{1}{2}c-2d\right)$$
$$-\Psi\left(1+\frac{1}{2}c-d\right), \tag{7}$$

$$\frac{\partial^n}{\partial c^n}A=\sum_{r=0}^{n-1}\binom{n-1}{r}\frac{\partial^r}{\partial c^r}A\cdot\frac{\partial^{n-r-1}}{\partial c^{n-r-1}}B \tag{8}$$

$$\frac{\partial^{n-r-1}}{\partial c^{n-r-1}}B = (-1)^{n-r}(n-r-1)!\Big\{\zeta(n-r,1+c-2d)-\zeta(n-r,c) + \frac{1}{2^{n-r}}\zeta\left(n-r,\frac{1}{2}c\right) - \frac{1}{2^{n-r}}\zeta\left(n-r,1+\frac{1}{2}c-2d\right) - \frac{1}{2^{n-r-1}}\zeta\left(n-r,1+\frac{1}{2}c-d\right)\Big\}. \quad (9)$$

Proof. Setting z = 1, a1 = a2 = d and b1 = b2 = 1 +c−d in (10) gives after simple manipulations

$$\cdot\int_0^1\int_0^1 x^{a_1-1}(1-x)^{b_1-a_1-1}y^{a_2-1}(1-y)^{b_2-a_2-1}(1-xyz)^{-c}\mathrm{d}x\,\mathrm{d}y, \quad (10)$$

$$\int_0^1\int_0^1 x^{d-1}(1-x)^{c-2d}y^{d-1}(1-y)^{c-2d}(1-xy)^{-c}\mathrm{d}x\,\mathrm{d}y = \frac{\Gamma^2(d)\,\Gamma^2(1+c-2d)}{\Gamma^2(1+c-d)}\,{}_3F_2\left[\begin{matrix} c, & d, & d; \\ 1+c-d, & 1+c-d; & \end{matrix}\ 1\right] \quad (11)$$

Now, evaluating the 3F2 appearing in the right-hand side of (11) with the help of Dixon's summation theorem (12) by taking a = c, b = c = d and after some simplifications, we easily arrive at the following interesting double integral formula:

$${}_3F_2\left[\begin{matrix} a, & b, & c; \\ 1+a-b, & 1+a-c; & \end{matrix}\ 1\right] = \frac{\Gamma\left(1+\frac{1}{2}a\right)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma\left(1+\frac{1}{2}a-b-c\right)}{\Gamma(1+a)\Gamma\left(1+\frac{1}{2}a-b\right)\Gamma\left(1+\frac{1}{2}a-c\right)\Gamma(1+a-b-c)}. \quad (12)$$

$$\int_0^1\int_0^1 x^{d-1}(1-x)^{c-2d}y^{d-1}(1-y)^{c-2d}(1-xy)^{-c}\mathrm{d}x\,\mathrm{d}y = \frac{\Gamma^2(d)\,\Gamma\left(\frac{1}{2}c\right)\Gamma\left(1+\frac{1}{2}c-2d\right)\Gamma(1+c-2d)}{2\,\Gamma(c)\Gamma^2\left(1+\frac{1}{2}c-d\right)}, \quad (13)$$

provided that 0, −1 and

$$\Re(d)>0,\ \Re(c-2d)>-1 \text{ and } \Re(c-4d)>-2$$

ON THE NEW DOUBLE INTEGRAL TRANSFORM FOR SOLVING SINGULAR SYSTEM OF HYPERBOLIC EQUATIONS

Nonlinear equations are of great importance to our contemporary world. Nonlinear phenomena have important applications in applied mathematics, physics, and issues related to engineering. Despite the importance of obtaining the exact solution of nonlinear partial differential equations in physics and applied mathematics there is still the daunting problem of finding new methods to discover new exact or approximate solutions. In the recent years, many authors have devoted their attention to study solutions of nonlinear partial differential equations using various methods. Among these attempts are the Adomian decomposition method, homotopy perturbation method, variational iteration method, Laplace variational iteration method differential transform method, Elzaki transform, Laplace, double Laplace transforms and projected differential transform method. Many analytical and numerical methods have been proposed to obtain solutions for nonlinear PDEs with fractional derivatives, such as local fractional variational iteration method, local fractional Fourier method, Yang-Fourier transform and Yang-Laplace transform and other methods. Two Laplace variational iteration methods are currently suggested by Wu. In this paper, we will introduce the new method depends on double new integral transform (double Elzaki transform), and it will be employed in a straightforward manner. Also, we study in this paper the combination of this new transform and the new method to solve the singular system of hyperbolic equations. This approach can be taken functions with discontinuities as well as impulse functions effectively. Elzaki transform, henceforth designated by the operator E [.], is defined by the integral equation,

$$E\left[\Omega(t)\right] = T(\beta) = \beta^2 \int_0^\infty \Omega(\beta t) e^{-t} dt \qquad (14)$$

By analogy with the double Laplace transform, we shall denote the double Elzaki transform.

New double integral transform

In this and analogy with the double Laplace transform, we will denote the new double transform. Also, in this paper, we will see the importance of this new double transform and its effectiveness in solving some differential equations.

Definition 3. Let Ω(x, t), t, x ∈ R +, be a function which can be expressed as a convergent infinite series, then its new double integral transform given by

$$E_2\left[\Omega(x,t),\alpha,\beta\right] = T(\alpha,\beta) = \alpha\beta \int_0^\infty \int_0^\infty \Omega(x,t) e^{-\left(\frac{x}{\alpha}+\frac{t}{\beta}\right)} dx dt, \quad x,t > 0 \qquad (15)$$

where α, β are complex values. To find the solution of the singular system of hyperbolic equations by the combination of new double transform and the new method, first we must find the new double transform of partial derivatives as follows:

$$E_2\left[\frac{\partial\Omega}{\partial x}\right] = \frac{1}{\alpha}T(\alpha,\beta) - \alpha T(0,\beta), \qquad E_2\left[\frac{\partial^2\Omega}{\partial x^2}\right] = \frac{1}{\alpha^2}T(\alpha,\beta) - T(0,\beta) - \alpha\frac{\partial T(0,\beta)}{\partial x},$$

$$E_2\left[\frac{\partial\Omega}{\partial t}\right] = \frac{1}{\beta}T(\alpha,\beta) - \beta T(\alpha,0), \qquad E_2\left[\frac{\partial^2\Omega}{\partial t^2}\right] = \frac{1}{\beta^2}T(\alpha,\beta) - T(\alpha,0) - \beta\frac{\partial T(\alpha,0)}{\partial t},$$

$$E_2\left[\frac{\partial^2\Omega}{\partial x\partial t}\right] = \frac{1}{\alpha\beta}T(\alpha,\beta) - \frac{\beta}{\alpha}T(\alpha,0) - \frac{\alpha}{\beta}T(0,\beta) + \alpha\beta T(0,0). \qquad (16)$$

Proof.

$$E_2\left[\frac{\partial\Omega}{\partial x}\right] = \alpha\beta \int_0^\infty \int_0^\infty e^{-\left(\frac{x}{\alpha}+\frac{t}{\beta}\right)} \frac{\partial}{\partial x}\Omega(x,t) dx dt = \beta \int_0^\infty e^{-\frac{t}{\beta}} \left\{\alpha \int_0^\infty e^{-\frac{x}{\alpha}} \frac{\partial}{\partial x}\Omega(x,t) dx\right\} dt$$

The inner integral gives

$$\frac{1}{\alpha}T(\alpha,t) - \alpha\Omega(0,t).$$

and then:

$$E_2\left[\frac{\partial\Omega}{\partial x}\right] = \frac{\alpha}{\beta}\int_0^\infty e^{-\frac{t}{\beta}} T(\alpha,t) dt - \alpha\beta \int_0^\infty e^{-\frac{t}{\beta}} \Omega(0,t) dt = \frac{1}{\alpha}T(\alpha,\beta) - \alpha T(0,\beta)$$

Also,

$$E_2\left[\frac{\partial \Omega}{\partial t}\right]=\frac{1}{\beta}T(\alpha,\beta)-\beta T(\alpha,0)$$

We can prove the formulas mentioned in (16) easily by using the same method.

CONCLUSION

This paper examines the convergence of the new double transform, and explain the effectiveness and ease of the method used to solve the singular system of linear and nonlinear hyperbolic equations, as we obtained the exact solutions using only one step. Integral transform methods are one of the most important methods which have been used recently to solve partial differential equations. Therefore, many phenomena in mathematical physics, engineering and sciences fields can be modeled by mathematical equations written in terms of partial differential equations. Double integral is mainly used to find the surface area, and it is denoted using '∫∫'. We can easily find the area of a rectangular region by double integration. If we know simple integration, then it will be easy to solve double integration problems. So, first of all, we will discuss some basic rules of integration.

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