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NUMERICAL MODELING OF NONLINEAR QUASI-LINEAR HEAT TRANSFER PROBLEM

Ikromov Akmal Olimovich
Master of Applied Mathematics, Termiz State University

Abstract
Creation of effective calculation methods for solving non-linear and quasi-linear partial differential equations is one of the urgent issues in the direction of computing technologies. It is important to solve the quasi-linear heat transfer equation in the form of linear, quadratic and cubic functions of the heat transfer coefficient temperature using different iteration schemes. In the same way, it is envisaged to justify the efficiency of the used methods by the number of arithmetic operations, to study the number of arithmetic operations of the methods depending on the non-linear parameter.

Keywords: Implicit scheme, implicit iteration scheme, number of iterations, number of arithmetic operations, number of grid layers, grid steps.

Introduction
Consider the following boundary value problem for the heat transfer equation with nonlinear coefficients

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}(k(u)\frac{\partial u}{\partial x})+f(u)\quad ,0<x<1\,,0<t\leq T, \qquad (1)$$

$$u(x,0)=u_0(x)\ , \qquad 0\leq x\leq 1, \qquad (2)$$

$$u(0,t)=\mu_1(t)\ ,\quad u(1,t)=\mu_2(t),\quad 0\leq t\leq T, \qquad (3)$$

here $k(u)=k_0u$ - let the heat transfer coefficient be a non-linear function of temperature, $\sigma\geq 1$.
The differential problem (1)-(3) is continuous under consideration

$$\overline{D}=\{0\leq x\leq 1,\ \ 0\leq t\leq T\}$$

we introduce a separate mesh in the field

$$\overline{\omega}_{h\tau}=\left\{(x_i,t_j),\begin{matrix} x_i=ih\,,i=0,1,2,...,\,N,\quad h=1/N, \\ t_j=j\tau\,,j=0,1,2,...\,,M\,,\tau=T/M\end{matrix}\right\}.$$

We put the following differential problems corresponding to the differential problem on the differential mesh [1]: $\overline{\omega}_{h\tau}$

Scheme a):

$$\frac{\hat{y}_i-y}{\tau}=\frac{1}{h}\left[a_{i+1}(y)\frac{\hat{y}_{i+1}-\hat{y}_i}{h}-a_i(y)\frac{\hat{y}_i-\hat{y}_{i-1}}{h}\right]+f(y_i),\quad \begin{matrix}0<i<N\ , \\ 0\leq j<M,\end{matrix}$$

$$y_i^0=u_0(x_i),\qquad 0\leq i\leq N, \qquad (4)$$

$$y_0^{j+1}=\mu_1(t_{j+1})\ ,\qquad y_N^{j+1}=\mu_2(t_{j+1})\ ,\qquad 0\leq j<M.$$

Scheme b):

$$\frac{\hat{y}_i - y}{\tau} = \frac{1}{h}\left[a_{i+1}(\hat{y})\frac{\hat{y}_{i+1} - \hat{y}_i}{h} - a_i(\hat{y})\frac{\hat{y}_i - \hat{y}_{i-1}}{h}\right] + f(\hat{y}_i)\ , \quad 0 < i < N,\ 0 \le j < M,$$

$$y_i^0 = u_0(x_i)\ , \quad 0 \le i \le N, \qquad (5)$$

$$y_0^{j+1} = \mu_1(t_{j+1})\ , \quad y_N^{j+1} = \mu_2(t_{j+1})\ , \quad 0 \le j < M.$$

To solve these different problems, we introduce the progonka method.

$$\hat{y}_i - \frac{\tau}{h}\left[a_{i+1}(y)\frac{\hat{y}_{i+1} - \hat{y}_i}{h} - a_i(y)\frac{\hat{y}_i - \hat{y}_{i-1}}{h}\right] = y_i + f(y_i)\tau;$$

$$\hat{y}_i - \frac{\tau}{h^2}a_{i+1}(y)\hat{y}_{i+1} + \frac{\tau}{h^2}a_{i+1}(y)\hat{y}_i + \frac{\tau}{h^2}a_i(y)\hat{y}_i - \frac{\tau}{h^2}a_i(y)\hat{y}_{i-1} = y_i + \tau f(y_i);$$

$$-\frac{\tau}{h^2}a_i(y)\hat{y}_{i-1} + (1 + \frac{\tau}{h^2}(a_{i+1}(y) + a_i(y))\hat{y}_i - \frac{\tau}{h^2}a_{i+1}(y)\hat{y}_{i+1} = y_i + \tau f(y_i);$$

Now multiply by (-1) and we get:

$$\frac{\tau}{h^2}a_i(y)\hat{y}_{i-1} - (1 + \frac{\tau}{h^2}(a_{i+1}(y) + a_i(y))\hat{y}_i + \frac{\tau}{h^2}a_{i+1}(y)\hat{y}_{i+1} = -(y_i + \tau f(y_i));$$

In scheme a) and b), $\hat{y}_i = y_i^{j+1}$, $y_i = y_i^j$ and $a_i(\vartheta) = a(\vartheta_{i-1}, \vartheta_i)$ the coefficients can be calculated by any of the following formulas:

$$a_i(\vartheta) = 0{,}5[k(\vartheta_{i-1}) + k(\vartheta_i)],\ a_i(\vartheta) = k\left(\frac{\vartheta_{i-1} + \vartheta_i}{2}\right),\ a_i(\vartheta) = \frac{2k(\vartheta_{i-1})k(\vartheta_i)}{k(\vartheta_{i-1}) + k(\vartheta_i)}.$$

The accuracy of calculating the temperature wave $a_i(\vartheta)$ strongly depends on how the coefficients are calculated.

A theoretical comparison of schemes a) and b) was made in [1], and it was noted that since scheme b) is not linear, it is appropriate to use the following iteration process to solve it

$$\frac{\overset{(s+1)}{y}_i - y_i}{\tau} = \frac{1}{h}\left[a_{i+1}(\overset{(s)}{y})\frac{\overset{(s+1)}{y}_{i+1} - \overset{(s+1)}{y}_i}{h} - a_i(\overset{(s)}{y})\frac{\overset{(s+1)}{y}_i - \overset{(s+1)}{y}_{i-1}}{h}\right] + f(\overset{(s)}{y}_i), \quad 0 < i < N,\ 0 \le s < 3\ ,\ 0 \le j < M,$$

$$y_i^0 = u_0(x_i), \quad 0 \le i \le N, \qquad (6)$$

$$\overset{(s+1)}{y}_0 = \mu_1(t_{j+1})\ , \quad \overset{(s+1)}{y}_N = \mu_2(t_{j+1}), \quad 0 \le j < M.$$

This scheme $\overset{(s+1)}{y}$ is linear with respect to .

At first glance, since scheme a) does not require iteration, using it seems preferable to using scheme b) which requires iteration. However, practical calculations show that scheme b) is effective.

Therefore, $k(u) = k_0 u \quad , \sigma = 1$ it is of great practical importance to compare the effectiveness of schemes a) and b) in the case where the heat transfer coefficient is a non-linear function of temperature. The authors are not aware of any research work carried out in this direction.

It is known that the number of arithmetic operations is considered as the main indicator in evaluating the effectiveness of arbitrary numerical methods. In this article, the efficiency of schemes a) and b) $k(u) = k_0 u \quad , \sigma = 1$. is compared in terms of the number of arithmetic operations when the heat transfer coefficient is represented, and it is shown that scheme b) is a very efficient method.

The result of the calculation experiment shows that it is necessary to choose $\sigma = 1$ a very small time step in order for the parameter to reach a certain accuracy according to scheme a) τ , which, in turn, leads to a sharp increase in the number of arithmetic operations. It is shown that only three iterations per layer interval in time are enough to ensure a certain accuracy according to scheme b), as a result of which it is shown that a significant reduction in the number of arithmetic operations can be achieved.

It should be noted that differential schemes (4) and (6) are solved by the progonka method. It is known that 8N arithmetic operations are used to perform the propelling method in one layer, where N is the number of mesh nodes.

The number of arithmetic operations required to implement scheme a) is equal $Q_1 = 8N * N1$ to , the number of these operations is equal to for a different scheme b) $Q_2 = 8N * IT * N2$, where $8N$ the number of operations of the running method, $N1$ and , $N2$ respectively, according to the time in schemes a) and b) is the number of layers, IT and the scheme consists of the number of iterations to be performed in one layer in b).

Differential problem (1)-(3) is being considered

$$\overline{D} = \{0 \le x \le 1, \;\; 0 \le t \le T\}$$

we introduce the following differential mesh in the field

$$\overline{\omega}_{h\tau} = \left\{ (x_i, t_j), \begin{matrix} x_i = ih, i = 0,1,2,..., N, \quad h = 1/N, \\ t_j = j\tau, j = 0,1,2,\dots, M, \tau = T/M \end{matrix} \right\} .$$

To conduct a calculation experiment, we select the parameters of the problem as follows:

$$N = 50 , \; M = 6, \; T = 0.6 , \; k(u) = k_0 u^{\sigma} , \;\; \sigma = 1$$

$\sigma = 1$ the case $k(u) = k_0 u$ where the heat transfer coefficient is a linear function of temperature. Let the values for the steps of the mesh $h = 0.02$ and for a) scheme $\tau = 0.02$ and b) $\tau = 0.05$ be selected for the scheme. A calculation experiment was carried out by scheme a) and b) and the obtained results are presented in table 1. $\sigma = 1$ and when the mesh steps are chosen as above, the number of mesh layers for a) scheme will be N1=30, and for b) scheme N2=12.

Table 1

t_j / x_i		0	0.1	0.2	0.3	0.4	0.5	0.6
i=0	a)	0	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000
	b)	0	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000
i = 10	a)	0.0400	0.1296	0.3061	0.5654	0.8125	1.0118	1.1712
	b)	0.0400	0.1424	0.3270	0.5596	0.7802	0.9679	1.1253
i = 20	a)	0.1600	0.3355	0.5880	0.8468	1.0884	1.2977	1.4714
	b)	0.1600	0.3433	0.5785	0.8208	1.0482	1.2488	1.4205
i = 30	a)	0.3600	0.6237	0.8694	1.0847	1.2857	1.4706	1.6331
	b)	0.3600	0.6114	0.8442	1.0545	1.2503	1.4301	1.5909
i = 40	a)	0.6400	0.9053	1.0849	1.2436	1.3955	1.5416	1.6777
	b)	0.6400	0.8866	1.0654	1.2239	1.3744	1.5181	1.6530
i = 50	a)	1.0000	1.1000	1.2000	1.3000	1.4000	1.5000	1.6000
	b)	1.0000	1.1000	1.2000	1.3000	1.4000	1.5000	1.6000

Different solutions of scheme a) and scheme b) presented in table 1 are graphically compared in

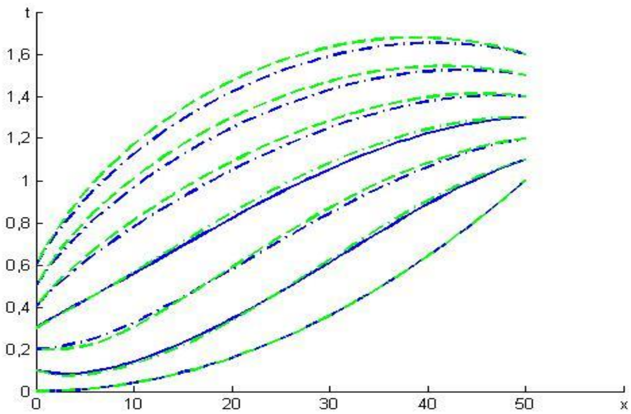

Figure 1.

Figure 1. A graphical comparison of the values in Table 1 obtained by the schemes, where scheme a) is a continuous line, scheme b) is a dotted continuous line.

Conclusion

It can be seen from table 1 and figure 1, obtained as a result of the calculation experiment, that the results obtained according to schemes a) and b) are slightly different.

1. Numerical solutions of the quasi-linear heat transfer equation when the heat transfer coefficient is in the form of a non-linear function of temperature were determined by explicit and implicit iteration schemes.
2. Undisclosed and undisclosed iteration schemes were compared according to the number of arithmetic operations, formulas for calculating the number of arithmetic operations were derived.
3. It was shown that the implicit iteration scheme is very effective in solving the given differential problem.

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