NOVATEUR PUBLICATIONS
JournalNX- A Multidisciplinary Peer Reviewed Journal
ISSN No: 2581 - 4230
VOLUME 10, ISSUE 4, April -2024

SOLUTION OF THE QUASI-LINEAR HEAT TRANSFER PROBLEM WITH PIECEWISE CONSTANT COEFFICIENTS

Qodirova Dilnoza Abdusalim qizi
Master of Applied Mathematics, Termiz State University

Abstract
In the article, the boundary value problem for second-order parabolic type equations, the approximation of differential schemes for the problem of non-stationary heat transfer and the maximum principle were studied. The flat approximation of the selected problem was checked. In calculus, we come across different differential equations, including different forms of partial differential equation, one of which is quasi-linear partial differential equation. Differential equations with partial derivatives of one or more dependent variables with several independent variables are called differential equations with partial coefficients.

Keywords: Implicit scheme, implicit iteration scheme, number of iterations, number of grid layers, grid steps.

Introduction
Let the following initial-boundary problem be set for the quasi-linear heat transfer problem with piecewise constant coefficients:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(k(x)\frac{\partial u}{\partial x}\right),\ a<x<b,\ 0\le t\le T, \tag{1.1}$$

$$\begin{aligned} u(a,t)&=0,\\ u(b,t)&=0, \end{aligned} \tag{1.2}$$

$$u(x,0)=u_0(x). \tag{1.3}$$

In the future different scheme in application of the differential problem (1.1). the following in appearance from writing used :

$$\frac{\partial u}{\partial t}=\frac{\partial k(x)}{\partial x}\cdot\left(\frac{\partial u}{\partial x}\right)+k(x)\frac{\partial^2 u}{\partial x^2}.$$

[a, b] to the cross section net we enter : $\varpi_h=\left\{x_i=ih, i=0,1,2,\dots,N,\ h=\frac{b-a}{N}\right\}$ and equation coefficient k(x) is considered net in pieces different immutable k_i values acceptance let him do it and in this

$$k(x)=\begin{cases} k_1, x_0<x<x_1,\\ k_2, x_1<x<x_2,\\ k_3, x_2<x<x_3,\\ \dots\dots\dots\dots\dots\dots\\ k_M, x_{M-1}<x<x_M, \end{cases}$$

to values have let it be

Different schemes

Of upper boundary value problem (1.1)-(1.3) by finite difference method is considered.
Watching

$$D=\{a\le x\le b,\ 0\le t\le T\}$$

in the field, the differential mesh is introduced as follows:

$$\overline{\omega}_{h\tau}=\left\{(x_i,t_j),\ \begin{matrix} x_i=ih\,, i=0,1,2,\dots,\ N, \quad h=(b-a)/N, \\ t_j=j\tau\,, j=0,1,2,\dots,M\,,\tau=T/M \end{matrix}\right\}.$$

Problem (1.1)-(1.3) is approximated using a pure implicit difference scheme. As a result, a differential problem of the following form corresponding to the differential problem (1.1)-(1.3) is set:

$$\frac{y_i^{j+1}-y_i^j}{\tau}=\frac{1}{h}\left[\frac{k_{l+1}+k_i}{2}\frac{y_{i+1}^{j+1}-y_i^{j+1}}{h}-\frac{k_i+k_{i-1}}{2}\frac{y_i^{j+1}-y_{i-1}^{j+1}}{h}\right],\ \begin{matrix} i=1,2,\dots N-1, \\ j=0,1,\dots M-1, \end{matrix} \tag{1.4}$$

$$y_0^{j+1}=0,\ y_N^{j+1}=0,\ j=0,1,\dots M-1, \tag{1.5}$$

$$y_i^0=u_0(x_i),\ i=0,1,\dots N. \tag{1.6}$$

Equation (1.4) is reduced to the following form in order to use the progonka method in solving the differential problem (1.4)-(1.6):

$$Ay_{i-1}^{j+1}-Cy_i^{j+1}+By_{i+1}^{j+1}=-F_i, \tag{1.7}$$

in this

$$A=\frac{(k_i+k_{i-1})\tau}{2h^2},\ B=\frac{(k_{i+1}+k_i)\tau}{2h^2}\,,\ C=1+\frac{((k_{i+1}+k_i)+(k_i+k_{i-1}))\tau}{2h^2},\ F_i=y_i^j;$$

differential equation (1.7) is solved numerically by applying the progon method with conditions (1.5)-(1.6).

The algorithm of the progonka method will look like this:

$$\overset{(\to)}{\alpha}_{i+1}=\frac{B}{C-A\alpha_i},\qquad \overset{(\to)}{\beta}_{i+1}=\frac{A\beta_i+F_i^{\,j}}{C-A\alpha_i},\ i=1,2,\dots,N-1, \tag{1.8}$$

$$y_N^{j+1}=0,\qquad y_i^{j+1}=\alpha_{i+1}y_{i+1}^{j+1}+\beta_{i+1},\ i=N-1,\dots 1,0,\ j=0,1,\dots,M-1, \tag{1.9}$$

Finding the initial values for the coefficients of the race to the expression (1.8). $i=0$ by putting , the following equation is formed:

$$y_0^{j+1}=\alpha_1 y_1^{j+1}+\beta_1. \tag{1.10}$$

Comparing the equation (1.10) with the boundary condition (1.5) , α_1 and β_1 the following expressions are formed for:

$$y_0^{j+1} = \alpha_1 y_1^{j+1} + \beta_1 = 0,\ \alpha_1 = 0,\ \beta_1 = 0. \quad (1.11)$$

expression (1.8) is the direct path of the progon method, and expression (1.9) is the inverse path of the progon method.

Differential problem (1.1)-(1.3) of time $t = 0$ in value $u(x,0) = u_0(x)$ initial a must with is considered The algorithm developed above using the differential scheme is applied to the calculation of the initial-boundary problem for the quasi-linear heat transfer equation (1.1)-(1.3) with piecewise constant constant coefficients.

In this case, the initial solution of the problem is considered in this form:

$$u(x,0) = \exp\left[-\frac{k_i x^2}{4t_0}\right], \quad (1.12)$$

Immutable t_0 determines the half-length of the original distribution: t_0 the smaller it is, the shorter it is. Numerical calculations basically of parameters the following in values take went to : $t_0 = 0.15$, $\tau = 0.01$. Integration interval $[-1,1]$ choosing that received Initial of distribution given half length for function (1.12) is marginal at points 10^{-12} accuracy with to zero equal to will be That's it Since , Eq (1.12) follows borderline conditions with viewed :

$$u(\pm 1, t) = 0, \quad (1.13)$$

Calculation experiment $k_1 = 150,\ k_2 = 200,\ k_3 = 250,\ k_4 = 300$ and $[-1,1]$ cross section equal to to intervals when divided $N = 16, N = 32$ and $N = 64$ to nodes have has been different from the scheme used without $t = 40\tau$ layer take gone , this on the ground τ time according to net step

Table 1.1.

Different method , in this $k_1 = 150, k_2 = 200, k_3 = 250, k_4 = 300$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2429	0.5201	0.7972	0.5155	0.2336
N=32	0.2065	0.3934	0.5748	0.3666	0.1747
N=64	0.2060	0.4105	0.5314	0.3902	0.1654

Now heat permeability coefficient k_i net in the elements as follows is selected : $k_1 = 200,\ k_2 = 300,\ k_3 = 350,\ k_4 = 250$ and $[-1,1]$ cross section equal to to intervals divided in case count equipment take will go

Table 1.2 .

Different method , in this $k_1 = 200,\ k_2 = 300,\ k_3 = 350,\ k_4 = 250$

	-0.1	-0.05	0	0.05	0.1
N=16	0.2286	0.5416	0.8544	0.5398	0.2251
N=32	0.1408	0.3378	0.6223	0.3170	0.1223
N=64	0.1311	0. 3635	0.5367	0.3455	0.1063

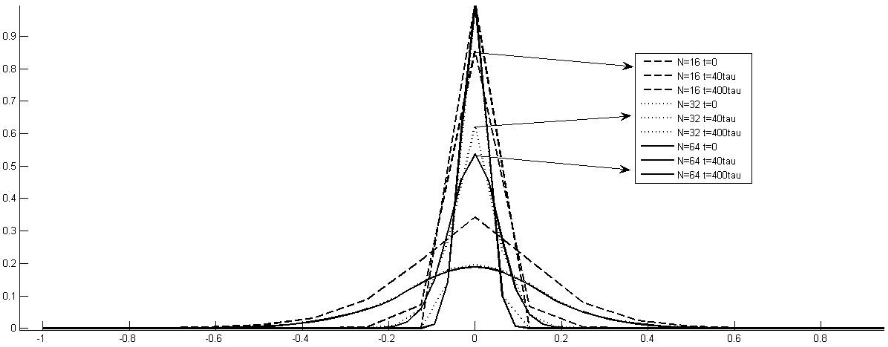

Figure 1.1. Different method with different in nets and each different time in layers calculated solution

Conclusions

1. Difference solutions for the quasilinear heat equation are determined, when the heat conductivity coefficient is a nonlinear function of temperature using an implicit and implicit iterative scheme.
2. The implicit and implicit iterative schemes are compared in terms of the number of arithmetic operations, formulas for calculating the numbers of arithmetic operations are derived.
3. The high efficiency of the implicit iterative scheme is shown in solving the formulated differential problem.

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