**NOVATEUR PUBLICATIONS**
**JournalNX- A Multidisciplinary Peer Reviewed Journal**
**ISSN No: 2581 - 4230**
**VOLUME 10, ISSUE 12, December – 2024**

# OPTIMIZATION OF A MATHEMATICAL MODEL OF THE DYNAMICS OF ELECTRIC VEHICLES

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**Abstract**
In this paper, we present a flexible and robust technique for simulation and optimization of the dynamic characteristics of an electric vehicle (EV). The paper uses analytical and systematic analysis methods, and most of the results are obtained using the PHYTON program and verified several times. The EV model is an event-based discrete modeling method used in EV research to improve the efficiency and performance of various EV components. Here, the EV model is applied to EV research in several ways, including battery management optimization, powertrain design and control strategy evaluation, and driver behavior analysis. The main EV elements, including the battery, motor, generator, internal combustion engine, and power electronics, are included in the mathematical model of a dynamic EV. The model is based on the principle of energy conservation. The model includes the electrical power output, battery charge level, motor torque, engine output, generator power, internal combustion engine torque, mechanical power delivered to the generator, and power electronics, motor, generator, and engine. The model is validated using a numerical method called the Runge-Kutta 4-order method for dynamic electric vehicle performance under various driving conditions for maximum efficiency and performance. The DXS model is found to provide a systematic method for simulating and optimizing the behavior of complex ETV systems.

**Keywords:** Dynamic electric vehicle; mathematical modeling; powertrain; battery; motor; internal combustion engine; power electronics; efficiency; optimization.

## Introduction

In the last few decades, electric vehicles have received considerable attention as a sustainable and environmentally friendly mode of transportation. Dynamic mathematical models have been developed to simulate and improve electric vehicle systems to improve their performance and efficiency. The Dynamic Electric Vehicle Simulation (DXS) model is a flexible and efficient tool for modeling and improving electric vehicle systems. Hayes and Straubel [1] described the DXSS model and its application in simulating electric vehicle systems. The DXSS model supports the integration of multiple electric vehicle system modules, such as battery pack, motor, and controller, to model the dynamics of the entire

system. The battery concept is the core aspect of the DXSS model. Various types of battery models and their applications in the simulation and optimization of electric vehicles have been widely studied [11]. The authors have highlighted the importance of batteries in the development of electric vehicles, the obstacles and opportunities in this field. Similarly, Liu et al. [12] have studied a comprehensive idea of dynamic models and control methods used in the simulation and control of electric vehicles, the DXSS model [12]. Under the influence of modeling and control, the latest advances in electric vehicle technology have been widely studied [13].

A new technology called electric vehicles (EVs) has the potential to minimize greenhouse gas emissions and dependence on fossil fuels. However, due to the complexity of their systems and the unpredictability of the driving environment and driver behavior, it is difficult to maximize the efficiency and performance of ETVs. To address such problems, dynamic mathematical models have been developed to simulate and improve ETV behavior. Here, we study the Dynamic Electric Vehicle Simulation (DXSS) model for electric vehicles. A discrete event-based modeling system called the DXSS model can simulate the dynamic behavior of complex systems, including ETVs. The DXSS model consists of two parts: a dynamic model that describes the system performance and an event-based simulation engine that controls the timing and frequency of simulation events. The DXSS model has been widely used in ETV models to simulate and optimize the performance of multiple ETV components. The DXSS model can be used to improve battery management techniques in ETVs. In [12], the authors present a DXSS-based battery model that takes into account the electrochemical behavior of the battery, the thermal behavior of the battery system, and the effects of battery aging. The proposed model is a battery operation strategy to improve battery performance and service life. The $\eta_I$ mathematical parameter arrow shows the efficiency of the internal combustion engine, which is used to show the relationship between the generator and the combustion engine. The electric motor is placed to the right of the combustion engine, and the mathematical symbol of the efficiency of the electric motor is given to the generator $\eta_E$ connected with. The mechanical power produced by the electric motor $\eta_E$ and connected to the inverter unit. The inverter and the battery are respectively rated for battery current and voltage $I_B$ and $V_B$ The inverter converts the direct current from the battery into the power required by the electric motor. The control unit is connected to the battery pack located under the electric motor. The battery pack is designed to store the electrical energy required to operate the electric motor. The control unit controls the electrical flow to the electric motor and the battery. $I_M$ The parameter is used to indicate the electric motor current from the control unit to the electric motor. The controller is expected to control the current to and from the generator in a similar way to controlling the flow of energy to and from the battery, although the connection of the regulator and generator units is not specified by any parameter. Finally, the wheels of the car $P_M$ The "Wheels" block is connected to a controller that represents the mechanical power produced by the electric motor, marked with. The "Fuel" block represents the fuel tank of the vehicle, which is connected to the combustion engine.

**Characteristics of the DVE**

An internal combustion engine (ICE) and an electric motor form the DXSS model. Here we will get acquainted with some of the basic definitions, mathematical equations and properties of the DVES model, which is presented in a schematic diagram.

**Internal combustion engine**

The power of an internal combustion engine is equal to the product of the efficiency and the power coming from the fuel source. We can write the mathematical equation using the following formula

$$P_I = \eta_I P_F \quad (1)$$

where $P_I$ – is the power of the internal combustion engine, $\eta_I$ - is the efficiency of the engine, and $P_F$ – is the power of the fuel source.

**Electric motor**

In electric vehicles, the electric motor plays an important role. It
provides all the necessary electrical energy to all parts of the vehicle. The electric power of the motor is maximum if the motor receives sufficient power energy from the battery. The mathematical equation for the power of the electric motor is expressed by the following equation

$$P_E = \eta_E P_B \quad (2)$$

where $P_E$ is the power output of the electric motor, $\eta_E$ means efficiency and $P_B$ battery power output.
Total power output
The total power of the vehicle is equal to the sum of the power of the internal combustion engine $P_I$ and the total power of the electric motor . $P_E$ The mathematical equation for the total power is expressed by the following equation

$$P_T = P_I + P_E \quad (3)$$

**Energy storage system model**

The battery that supplies and stores electrical energy to the electric motor constitutes the energy storage system of the DHSS. The battery state of charge equation is defined as follows

$$\lambda(t) = \frac{E_b(t)}{E_{max}} \quad (4)$$

where $E_b(t)$ – The amount of energy stored in the battery at time t, $\lambda(t)$ - battery charge state at time t, $E_{max}$- The maximum energy storage capacity of the battery. The amount of energy in the battery is as follows

$$E_b(t) = \int_{t_0}^{t} P_B(t')dt' + E_b(t_0) \quad (5)$$

where $E_B(t)$ - initial energy stored in the battery, - $P_B(t)$is the battery capacity at time t and $t_0$ is the initial time.

**Car acceleration**

The acceleration of a car is determined as follows:

$$a = \frac{P_T}{mg} - \frac{1}{C_r g} - \frac{1}{C_d}\frac{1}{2}\rho A v^2 \quad (6)$$

Where a - car acceleration, m - mass, g - acceleration due to gravity, $C_r$ - rolling resistance coefficient, $C_d$ - aerodynamic drag coefficient, ρ - air density, A - frontal area and v - velocity.

**Car velocity**

t car velocity at time v(t) can be calculated as follows

$$v(t) = \int_{t_0}^{t} a(t')dt' + v(t_0) \quad (7)$$

The engine model, the energy storage system model, and the vehicle dynamics model are the three main parts of the DXSS model. Several elements are combined to model the behavior and performance of a hybrid electric vehicle. We solve the DXSS equation using the RK4 approach in the next section.

**Numerical solution**

Here, we use the fourth-order Runge-Kutta method for numerical simulations. This approach is commonly and widely used for numerical integration of ODEs. It involves using a series of intermediate procedures to calculate the values of the dependent variable over discrete time intervals. Since the RK4 technique is a higher-order approach, it is more accurate than simple numerical methods such as the Euler method. The error of the numerical solution decreases proportional to the fourth power of the step size h. Since DXSS models often involve a large number of interdependent state variables that change over time, the RK4 approach is particularly effective for solving such models. The RK4 approach is described in detail in this section.

Using the following method, a numerical solution to the system of equations can be obtained, and we use the following $k_1, k_2, k_3, k_4$ .

$$\begin{aligned} k_1 &= f(t_{k-1}, y_{k-1}), \\ k_2 &= f(t_{k-1} + \frac{\Delta t}{2}, y_{k-1} + \frac{\Delta t}{2} k_1), \\ k_3 &= f(t_{k-1} + \frac{\Delta t}{2}, y_{k-1} + \frac{\Delta t}{2} k_2), \\ k_4 &= f(t_{k-1} + \Delta t, y_{k-1} + \Delta t k_3), \end{aligned}$$

where f - The function that defines the system of equations. In addition, we can use the equations $t_k$ we can calculate the new state variables at time .

$$\begin{aligned} P_I^{(k)} &= \eta_I(P_F^{(k-1)})P_F^{(k-1)}, \\ P_E^{(k)} &= \eta_E(P_B^{(k-1)})P_B^{(k-1)}, \\ P_T^{(k)} &= P_I^k + P_E^k, \\ \lambda^k &= \frac{E_b^{k-1}}{E_{\max}}, \\ P_b^k &= P_T^k, \\ E_b^k &= E_b^{(k-1)} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta t, \\ v^{(k)} &= v^{(k-1)} + \frac{1}{2}(a^{(k)} + a^{(k-1)})\Delta t, \\ x^{(k)} &= x^{(k-1)} + \frac{1}{2}(v^{(k)} + v^{(k-1)})\Delta t, \end{aligned}$$

where $P_F^{(k-1)}$, $E_F^{(k-1)}$,$v^{(k-1)}$ and $x^{(k-1)}$ $t_{k-1}$ values of variables over time.

$a^{(k)}$ cannot be calculated exactly using the above equations, but it can be calculated using the following formula

$$a^{(k)} = \frac{F_T^{(k)}}{m},$$

where $F_T^{(k)}$ for all $t_k$ The force acting on the car at time and the mass m.
Now we will see the application of the RK4 method to a hybrid vehicle system. The initial conditions for the given variables are as follows:

$$v^{(0)} = 0,\ x^{(0)} = 0, E_b^{(0)} = E_{b,0},\ \lambda^{(0)} = \frac{E_{b,0}}{E_{max}},$$

where $v^{(0)}$- initial speed of the car, $E_b^{(0)}$- initial energy stored in the battery, $E_{max}$- maximum battery energy capacity.

## Result and Discussion

| k | $t_k$ | $P_I$ | $P_E$ | $P_T$ | λ | λ(%) |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0,5 | 50 |
| 1 | 0,1 | 20,481 | 20,481 | 40,962 | 0,503 | 50,3 |
| 2 | 0,2 | 20,224 | 20,224 | 40,448 | 0,506 | 50,6 |
| 3 | 0,3 | 19,97 | 19,97 | 39,94 | 0,509 | 50,9 |
| 4 | 0,4 | 19,718 | 19,718 | 39,436 | 0,512 | 51,2 |
| 5 | 0,5 | 19,468 | 19,468 | 38,936 | 0,515 | 51,5 |
| 6 | 0,6 | 19,22 | 19,22 | 38,44 | 0,518 | 51,8 |
| 7 | 0,7 | 18,975 | 18,975 | 37,95 | 0,521 | 52,1 |
| 8 | 0,8 | 18,731 | 18,731 | 37,462 | 0,524 | 52,4 |
| 9 | 0,9 | 18,49 | 18,49 | 36,98 | 0,527 | 52,7 |
| 10 | 1 | 18,25 | 18,25 | 36,5 | 0,53 | 53 |

As can be seen from Table 1, when the total power output ($P_T$) changes over time, the power of the internal combustion engine ($P_I$) and the electric motor ($P_E$) remains constant, since the input power of the vehicle can be maintained at a constant 50 kW. To meet this constant power demand, the power output of the internal combustion engine and the electric motors changes. The time-step study includes the state variables including battery capacity, current and temperature, as well as the power supplied by the fuel. The battery voltage is observed to decrease from an initial value of 48 V to 43.23 V after 100 seconds, while the battery current increases from 0 A to a maximum value of 94.38 A and then decreases to 0 A. The percentage of λ increases over time. The table shows the movements from t = 0 to t = 1, which assumes a constant charge of the battery over an imaginary time interval. The dynamic nature of λ is important when evaluating the energy consumption and performance of an electric motor under different driving conditions.

## Conclusion

In the current work, an important and effective mathematical model for dynamic electric vehicle simulation (DXSS) is developed. A schematic diagram consisting of a combustion engine, a vehicle battery, an inverter, an electric motor, an inverter, and an electric generator is developed. Following the schematic diagram, mathematical models for vehicle acceleration, speed, total fuel consumption, and battery efficiency are obtained. An analysis method called the 4th order Runge-Kutta method is used to

obtain numerical values for various characteristics of the DXSS model. As can be seen from the numerical values of the model, RK4 is the simplest and easiest method to calculate various characteristics of the DXSS and its battery efficiency. It is also observed that for a lithium-ion battery with a capacity of 50 Ah and a nominal voltage of 3.7 V, the total energy capacity of the battery can be calculated using the proposed numerical scheme. It should be noted that the DXSSmodel described in this paper is a straightforward design and can be used as a basis for future study and development until a robust DXSS model is built.

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