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ALGORITHMIC ANALYSIS FOR AN INTRODUCTION TO FUZZY SET THEORY

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Abstract

Mathematical fuzzy set theory and fuzzy logic are generalizations of traditional set theory and classical formal logic. Originally a theoretical concept, it has now evolved into a fully-fledged decision-making technique. This paper provides a review of existing algorithms that utilize fuzzy logic to enhance image quality.

This article reviews the basic principles of fuzzy set theory, the principles of operation of fuzzy logic systems, and their real-life applications. It also analyzes complex decision-making processes and their effectiveness using fuzzy logic systems.

Introduction

Today, it is becoming increasingly clear that classical mathematical approaches are insufficient for the analysis and control of complex systems. Because many real-life processes do not have clear boundaries and involve uncertainties associated with human thinking and experience, classical set theory and traditional logic systems remain limited in effectively solving such problems.

Fuzzy set theory and fuzzy logic were proposed as a solution to these problems by Lotfi Zadeh in 1965, and were formed as a generalization of classical logic and set theory. This approach is closer to human thinking and is a convenient tool for modeling and controlling complex systems. Fuzzy logic and its basic principles are currently widely used in artificial intelligence, expert systems, automation, financial analysis, medical diagnostics, robotics, and many other fields . .

Mathematical support:

A key property of a fuzzy set is its membership function (MF). We denote it by $MF_x(x)$, which represents the degree of membership of the fuzzy set C. This concept is considered as a generalization of the characteristic function of traditional sets .

Thus, the fuzzy set C is a set of ordered pairs of the following form:

$$C = \{MF_x(x)/x\}$$

here $MF_x(x) \in [0,1]$

If $MF_x(x) = 0$ so, it doesn't belong to this element set at all.

- If $MF_x(x) = 1$ then this element fully belongs to the set C**..**
- $0 < MF_x(x) < 1$ If , the element is vaguely relevant.

This approach plays a key role in fuzzy set theory and extends classical set theory to model real-world uncertainties.

Let's explain this with a simple example. We will try to express the concept of "hot coffee" more clearly.

The vague set " Hot coffee"

Here **x** is the area of discussion and is taken as the temperature (°C) scale . Specifically, it ranges from 0 to 100°C. can change . The fuzzy set for the concept of "hot coffee" can be defined as follows:

$$C = \{0/0; 0/10; 0/20; 0.1/30; 0.3/40; 0.6/50; 0.8/60; 0.9/70; 1/80; 1/90; 1/100\}$$

What does this mean?

- Coffee at 60°C "Hot coffee" belongs to the set at a level of 0.80 .
- However, this value is based on personal intuition. It may depend on the person . 60°
- Correction: 60°C may be considered hot by one person but not hot enough by another,
- the ambiguity is: everyone has a different definition of "Hot Coffee."

Basic operations for fuzzy sets

As with traditional sets , logical operations are defined for fuzzy sets. The most important ones are:

1. Intersection (∩, AND)
2. Union (∪ , OR)

These operations are used to perform calculations on fuzzy sets and to analyze them logically. We will look at these operations in more detail in the next step .

two indefinite sets (indefinite "and"):

$$AB : MF_{AB}(x) = \min\left(MF_A(x), MF_B(x)\right)$$

two indefinite sets (indefinite "or"):

$$AB : MF_{AB}(x) = \max\left(MF_A(x), MF_B(x)\right)$$

In fuzzy set theory, a general approach to performing intersection, union, and addition operations has been developed, which are called triangle norms and conorms . The realizations of the intersection and union operations given above are the most common cases of the t-norm and t-conorm .

To describe fuzzy sets, the concepts of fuzzy and linguistic variables are introduced.

An uncertain variable is described by the set (N, X, A), where:

- **N —** variable name,
- **X** is the universal set (discussion domain),
- **A** is a fuzzy set in X.

linguistic variable can be fuzzy variables, meaning that the linguistic variable is at a higher level than the fuzzy variable. Each linguistic variable consists of:

- name;
- z values is also called the basic term set T. The elements of the basic term set represent the names of the ambiguous variables;
- universal set X ;
- syntactic rule G , which allows the creation of new terms using words from natural or formal languages;
- a semantic rule P that assigns a fuzzy subset of the set X to each linguistic variable value .

Let's consider a vague concept like "Stock price" . This is the name of a linguistic variable . We will form a basic term set for it . It will consist of three vague variables: "Low", "Medium", "High" , and we will

define the domain of discussion $X=[100;200]$ (in units) as . The last step is to construct the corresponding membership functions for each linguistic term in the basic term set T .
dozens of typical curves for defining membership functions . The most common are triangular , trapezoidal , and Gaussian. membership functions.
The triangular membership function is defined by three numbers (a, b, c) and its function at the point x is

$$MF(x)=\begin{cases}1-\dfrac{b-x}{b-a}, a\le x\le b\\ 1-\dfrac{x-b}{c-b}, b\le x\le c\\ 0, x\notin(a;c)\end{cases}$$

If (b−a) = (c−b) , a symmetric triangular membership function is formed, which can be defined as a single-valued function with two parameters from the triple (a, b, c).
Similarly, to define the trapezoid membership function, we need four (a, b, c, d):

$$MF(x)=\begin{cases}1-\dfrac{b-x}{b-a}, a\le x\le b\\ 1, b\le x\le c\\ 1-\dfrac{x-c}{d-c}, c\le x\le d\\ 0, x\notin(a;d)\end{cases}$$

If (b - a) = (d - c) , then the trapezoidal membership function has a symmetric appearance.

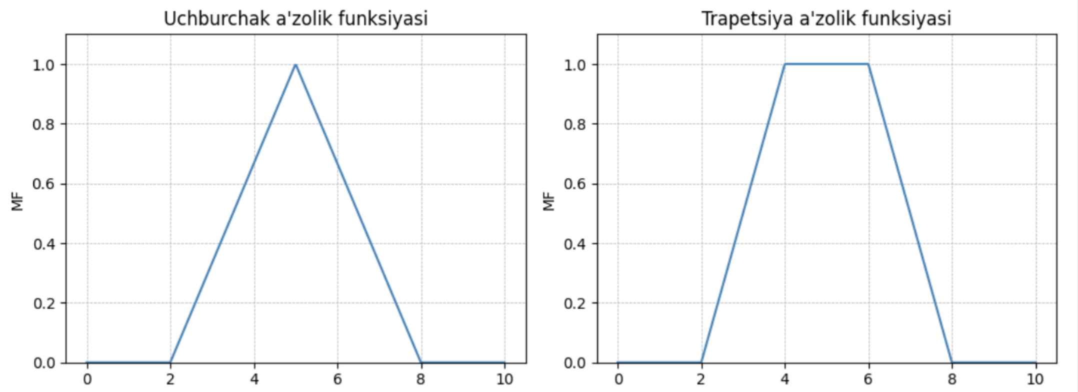

Figure 1. Typical piecewise-linear membership functions
The Gaussian-type membership function is expressed by the following formula:

$$MF(x)=\exp\left[-\left(\frac{x-c}{\sigma}\right)^2\right]$$

This function works with two parameters:
- c is the center of the fuzzy set ,

- σ — the parameter responsible for the slope (width) of the function.

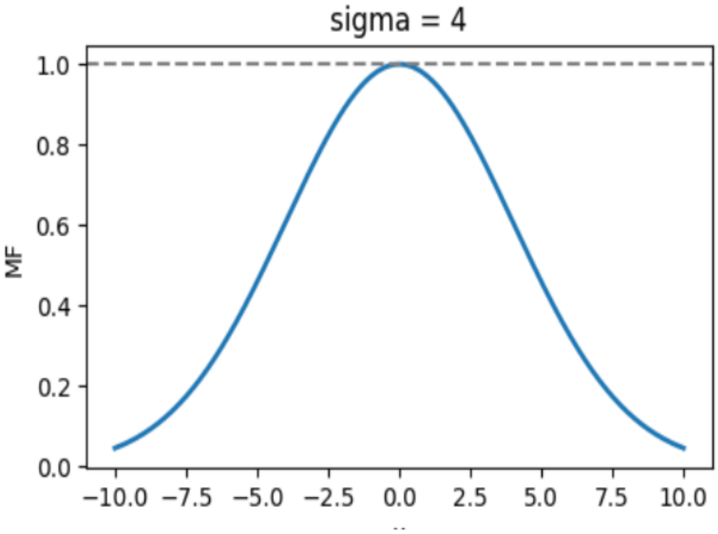

Figure 2. Gaussian membership function

The membership functions for each term belonging to the base term set T are usually depicted on a single graph.

Figure 3 shows an example of the linguistic variable "Stock Price" described above , while Figure 4 shows the formalization of the concept "Human Age" .

For example, for a 48-year-old person, the membership levels are as follows:

- "Youth" set — 0 ,
- "Average age" — 0.47 ,
- "Above average" — 0.20 .

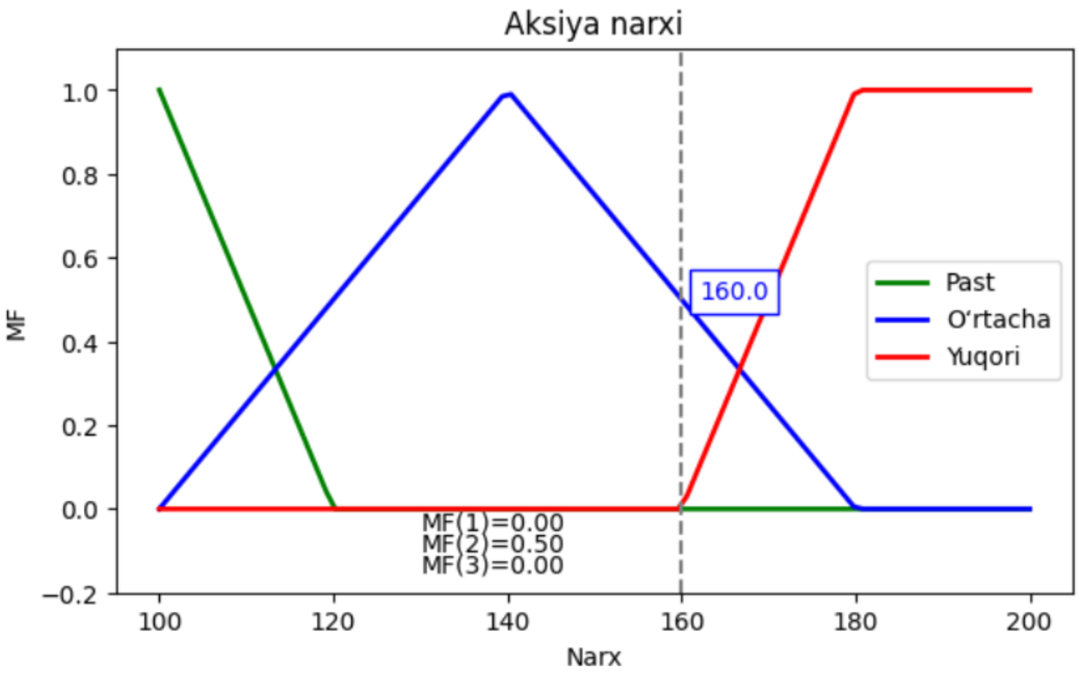

Figure 3. Description of the linguistic variable "Stock price"

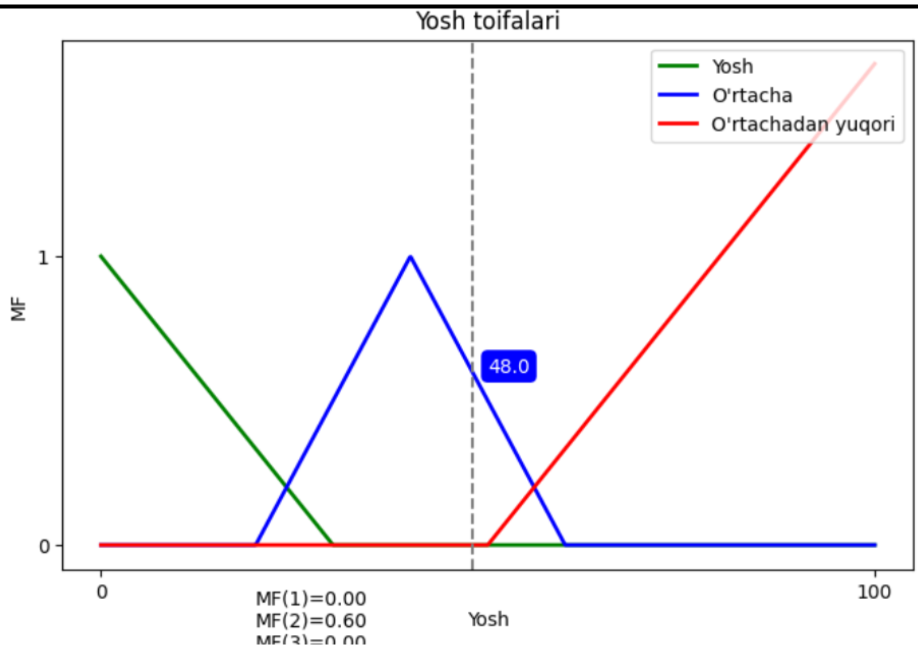

Figure 4. Description of the linguistic variable " age"

a linguistic variable rarely exceeds 7.

The result of fuzzy inference is a crisp output value y based on the given input values x_k , k = 1.. n based on calculated clear value y^* is taken .

Fuzzy inference algorithms differ mainly in the type of rules used , logical operations, and defuzzification method. Mamdani, Sugeno, Larsen, and Tsukamoto models of fuzzy inference have been developed.

Look at fuzzy inference using the Mamdani mechanism . This is the most common method of logical inference in fuzzy systems. It uses minimax composition of fuzzy sets . This mechanism involves the following sequential actions :

Fuzzification process: For each rule's left-hand side (conditional part), the function's fitness levels are determined. If the rule base consists of m rules , the fitness levels are determined by A_{ik} (xk) , where i = 1..m, k = 1..n.

First, the "cut" levels are determined for the left part of each rule:

1. Indeterminate inference: The level of "cut" is determined for the left part of each rule:

$$\alpha\, fa_i = \min(A_{ik}(x(x_k))$$

2. Then the "cut" membership functions are found:

$$B_i^*(y) = \min\left(\alpha_i, B_i(y)\right)$$

3. Composition

1. At this stage, the obtained cut functions are combined, for which the maximum composition of fuzzy sets is used:

$$MF(y) = \max(B_i^*(y))$$

where MF(y) is the membership function of the final fuzzy set .

4. Defuzzification (conversion to a specific value)

There are several methods for converting a fuzzy value into a precise value . For example, the center of gravity method or the centroid method:

$$y^* = \frac{\int yB(y)dy}{\int B(y)dy}$$

The geometric meaning of this approach is the center of gravity of the MF(y) curve . Figure 6 graphically shows the fuzzy inference process according to the Mamdani method for two input variables and two fuzzy rules (R1 and R2) .

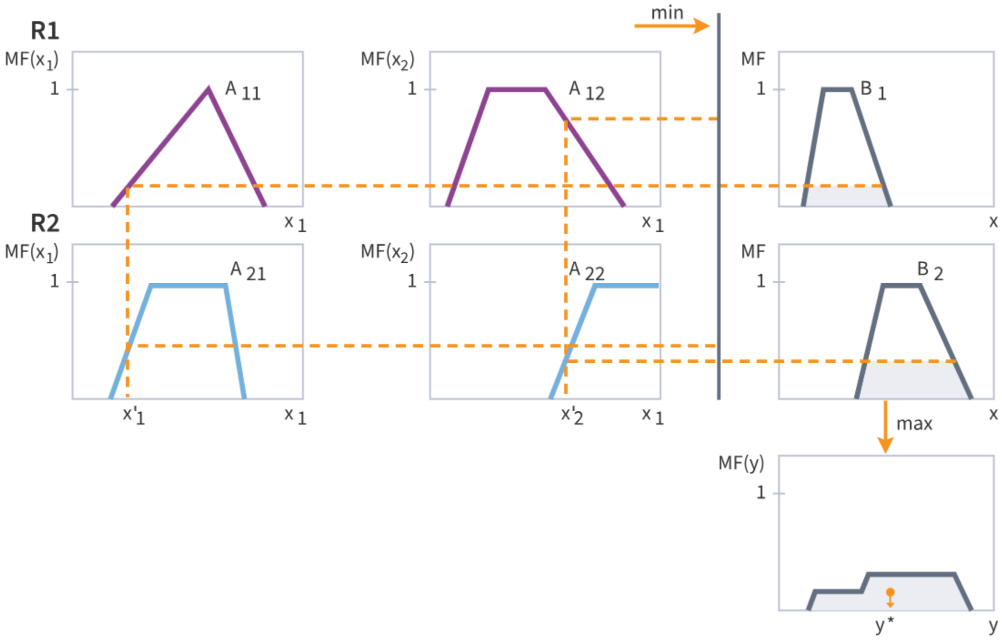

Figure 6. Scheme of fuzzy inference according to the Mamdani method

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