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DEVELOPMENT OF STUDENTS' MATHEMATICAL ABILITIES USING SEVEN COMPONENTS OF GEOMETRIC PROBLEMS

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Abstract
This article discusses the methodology of creating tasks that contribute to the development of mathematical abilities of students and pupils.

Keywords: Mathematical abilities, thinking, grasping, logic, generalization, reasoning, flexibility, rationality.

Introduction

The large-scale reforms carried out in our country, the processes of change and renewal in the education sector of the country, serve the goals of educating the younger generation, their comprehensive improvement, a generation with independent thinking. An example of this is innovations in various teaching methods aimed at strengthening the knowledge of young people and identifying their abilities. In this process, the problem of determining and developing mathematical abilities is an urgent task. The task of increasing the effectiveness and efficiency of education through the use of modern technologies, innovative pedagogical forms of education, and improving the quality of education by identifying and supporting capable youth is significant.

Experts in this field have published a number of works on this topic. They study various aspects of the problem of determining and developing students' mathematical abilities [1-6, 8-12].

It should be mentioned that in V.A.Krutetsky's book [1], from a scientific point of view, the components of mathematical abilities of pupils in grades 5-6 were studied and 9 types of components of mathematical abilities of pupils were developed. Utarov T.U. [2] introduced 12 components of mathematical abilities of students in grades 7-9.

Kostina E.A.[3] The structure of mathematical abilities was developed for university students. However, these works did not pay attention to the methodology of composing tasks for the development of components of mathematical abilities and their solutions.

Let's focus on the geometric problems of developing the components of students' mathematical abilities.

These are the following components.

1. The ability to grasp the formal structure of a task.
2. Logical mathematical thinking.
3. The ability to generalize mathematical material.
4. The reversibility of mathematical thinking.
5. The ability to curtail mathematical reasoning.
6. The ability of mathematical thinking flexibility.

7. The ability of rational mathematical thinking.

I. The ability to grasp the formal structure of a problem is the ability to extract the most useful information from a problem condition to solve it. The representation is considered to be a holistic image of this. Understanding its shape, its size, and the consistency of its color is of great importance in practice. The content of the presentation is determined by the tasks assigned to the person, as well as the reasons for his activity. In the process of perception through analysis, certain feelings form the receptive side of perception. The student's understanding of the text during the lesson includes perception by sight and hearing.

Spatial perception is the ability to extract from the conditions of a problem the most useful information for its solution, a necessary condition for determining the environment surrounding a person.It is a reflection of an objectively existing space and includes the perception of the shape of objects, their volume and relative position, their level, remoteness and orientation. The perception of senior students will be more focused and they will be able to manage them themselves. In the process of the teacher's purposeful guidance of students' activities, their perception will develop. During the lesson, students will acquire knowledge by listening to the teacher's oral explanation. The perception of the material presented orally is closely related to the peculiarities of the teacher's speech.

The development of the ability to grasp the formal structure of a task will be facilitated by the use of this task.

Task. A straight line perpendicular to the plane of the triangle is drawn through the center of the circle inscribed in the triangle. Prove that each point of this straight line is equidistant from the sides of the triangle.

Solution. Let A, B, C – be the points of tangency of the sides of the triangle with the circle, be the center of the circle, and be a point on the perpendicular (Fig. 1). S expresses the process of the student's spatial perception of the choice of the perpendicular point. Since the radius is perpendicular to the side of the triangle, then by the three-perpendicular theorem, the segment

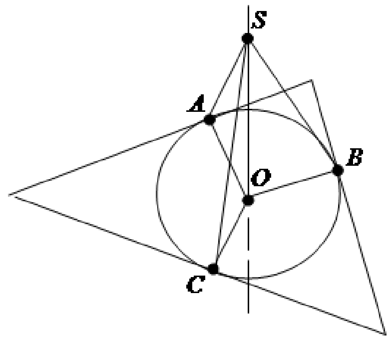

Fig.1.

SA there is a perpendicular to this side, and its length is the distance from the point to the side of the triangle. By the Pythagorean theorem $SA = \sqrt{AO^2 + OS^2} = \sqrt{r^2 + OS^2}$, where r – the radius of an inscribed circle. Similarly, we find $SB = \sqrt{r^2 + OS^2}$, $SC = \sqrt{r^2 + OS^2}$, that is, all distances from point S to the sides of the triangle are equal

This task will contribute to the development of the student's ability to understand the formal structure of space.

[7]- Tasks 2.2.6, 2.2.8, 2.2.17, in the collection of tasks, can be used as tasks to develop students' grasp of the formal structure of space.

II. The logic of mathematical thinking – the ability to correctly carry out consistent mathematical reasoning. Memorizing logical characteristics, proving theorems, and solving various problems begins with logical and linguistic analysis.

1) definition of mathematical expressions and their corresponding signs in the text; 2) definition of logical connections in the text; 3) finding open and hidden expressions of content and meaning; 4) getting rid of unnecessary model contents. The transition of mathematical thinking from a simple language to a logical mathematical language is one of the elements of knowledge. In other words, the student must understand the logical structure of the conversation, the transition from the language of formulas to symbolic notes and vice versa. Understanding the text and working logically on it, for example, in the presence of contradictory thoughts, requires intelligence and ingenuity from the student. When transmitting information, he should pay attention to the correct use of a particular language and related speech. This approach will give good results in the formation of students' logical culture.

When mastering the basic elements of logic, it is necessary: 1) correctly express constant mathematical thoughts or theorems 2) first of all, it is necessary to understand correctly, and then tell us what concepts we are studying, the conditions and conclusions of the theorem, the use of connectives such as "and" and "or", what conditions are being discussed; 3) the use of negation of mathematical conclusions of de Morgan laws in in the form of quantifiers and non-quantifiers; 4) to give students an idea of the methods of proving theorems; 5) when teaching geometry, explain the axiomatic structure of the basic logical principles of mathematical theories.

When developing logical thinking abilities in the study of mathematics, it would be advisable to use the following tasks

Task. Each segment contains at least one point.

Solution. Let A and B – the ends of the segment (fig.2). according to the axiom I_3 there is a point C outside the straight line AB. Let's take a point D on the line AC so that C is between A and D. This is possible by axiom II_4. This consideration will be the first step for students, choosing a point D that does not lie on the segment AB will be proof that students have logical thinking.

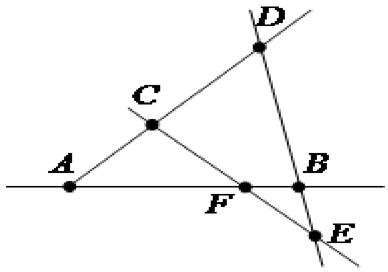

Fig.2.

Let's take the point E on the line BD so that B is between D and E. The straight line CE divides the plane into two half-planes. The points B and D are in the same half-plane, since the segment BD does not intersect the line CE , and the points A and D are in different half-planes, since the segment AD intersects the line CE (at point C). It follows that points A and B are in different half-planes, which means that the segment AB intersects the line CE. The intersection point of F is the point of the segment AB .

Let A, B, and C be three points that do not lie on the same straight line. A figure made up of three segments AB, BC, and CA is called a triangle, points A, B, and C are the vertices of the triangle, and segments AB, BC, and CA are the sides.

This type of task enables students to understand the axiomatic structure of the basic logical principles of mathematical theories in teaching geometry and, thus, develops students' logical thinking abilities. Tasks 2.2.3, 2.2.7, 2.2.14, 2.2.20 of the collection of tasks [7] can be used to develop students' logical thinking abilities using the example of the above tasks.

III. The ability to generalize mathematical material – the ability to see the similarities between different tasks, highlight the main thing in the solution method, and generalize the solution method. In psychology, generalization is defined as combining the basic properties of objects and events into one common object. In logic, this category is considered one of the foundations of the thinking process. It is used to search for the most important properties characteristic of a whole class of objects. The didactic essence of generalization is the identification of common, important properties and characteristics of the studied subject, as well as the formation and expression of concepts, laws that lead to an understanding of the studied subject.

To get a complete picture of the studied object and events, the generalization process begins with the activation of attention, thinking, memory and imagination. In the education system, the object of generalization can be the properties of objects, real events, features and properties, relationships, connections, processes. The more complex the object, the more difficult it is for the student to summarize the material.

At the same time, thinking makes it possible to identify basic concepts, characteristics, and relationships in the studied material. In the process of thinking, it is possible to identify common important properties and relationships of objects or events by comparison. The process of generalization, representing a complex work of thinking, includes representations, analysis, synthesis, comparison, memory and concepts, makes it possible to draw conclusions about advanced ideas of processes or events, to express certain patterns. When developing the ability to generalize, it would be advisable to use the following tasks.

Task. Prove that if the figure F_1 similar to figure F_2 , And the figure F_2 similar to figure F_3, The shapes F_1 and F_3 are similar.

Solution. Let X_1 and Y_1 – two arbitrary points of the shape F_1. Similarity transformation that translates the shape F_1 in F_2 , translates these points into points X_2 , Y_2 , for which $X_2Y_2 = k_1X_1Y_1$.

Similarity transformation that translates the shape F_2 in F_3, translates points X_2 , Y_2 To the points X_3 , Y_3 , for which $X_3Y_3 = k_2X_2Y_2$.

From the equalities

$$X_2Y_2 = k_1X_1Y_1,\ X_3Y_3 = k_2X_2Y_2$$

it follows that $X_3Y_3 = k_1k_2X_1Y_1$. Setting points X_1 ,Y_1 to the points X_2 , Y_2 X_2 and Y_2 divide them into dots X_3 , Y_3 and the conclusions drawn from them require their implementation through the ability of generalization. This in turn means replacing the shape F_1 on the figure F_3. Hence, the figures F_1 and F_3 – they are similar, which was required to be proved.

This process develops the student's ability to generalize.

Tasks 2.3.44, 2.3.47, 2.3.58 of the collection of problems [7] can be used to develop students' ability to generalize mathematical material as the above tasks.

IV. The reversibility of mathematical thinking - the ability to switch from forward to backward reasoning. This kind of ability is the ordering of a sequence of mathematical thoughts and, if necessary, using them in the opposite order. This thought is to use a sequence of conditions leading to a certain thought in their opposite sense.

To develop the reversibility of mathematical thinking, it is advisable to use tasks of this type.

Task. Prove that the symmetry transformation with respect to a straight line is motion.

Solution. Let's take this straight line as the axis of the Cartesian coordinate system (Fig. 3). Let be an arbitrary point $A(x;y)$ shapes F moves to a point $A'(x';y')$ shapes F'.

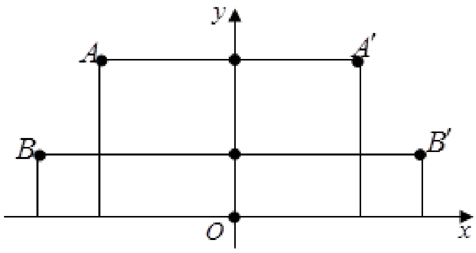

Fig.3.

From the definition of symmetry with respect to a straight line, it follows that the points A and A' ordinates are equal, and abscissas differ only by a sign:

$x' = -x$.

Let's take two arbitrary points $A(x_1;y_1)$ and $B(x_2;y_2)$. They will move to points $A'(-x_1;y_1)$ and $B'(-x_2;y_2)$.

We have:

$$AB^2 = (x_2 - x_1) + (y_2 - y_1),$$
$$A'B'^2 = (-x_2 + x_1) + (y_2 - y_1).$$

From this it can be seen that $AB = A'B'$. This means that the transformation of symmetry with respect to a straight line is motion.

This task allows students to organize their thoughts and, if necessary, use them in reverse, which helps them develop the ability of opposite mathematical thinking.

Tasks 2.3.6, 2.3.12, 2.3.28, 2.3.30 of the collection of tasks [7] can be used to develop students' reversibility of mathematical thinking using the example of the above tasks.

V. The ability to curtail mathematical reasoning - the ability to spontaneously skip intermediate statements in the process of solving a problem without leading to errors. Mental discussion, the ability to think. To develop the ability to collapse mathematical reasoning, use the following task

Task. Prove that two different planes can be drawn through a straight line.

Solution. Let a – this straight line (fig.4.). According to the axiom I_3 There is point A, not lying in a straight line a. By virtue of problem 2, a plane can be drawn through a straight line and A point, let us denote it α_1. According to the axiom I_8 there is a point B that does not lie in the plane α_1. Let's draw a plane through the line a and point B α_2.

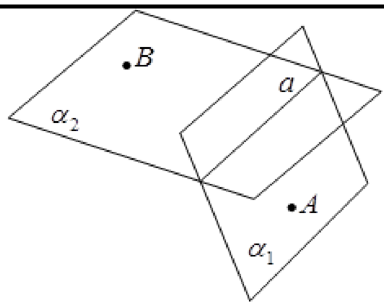

Fig.4.

Planes α_1 and α_2 They are different because point B of plane α_2 does not lie on plane α_1.
This task helps students to develop mathematical thinking.
Tasks 2.1.4, 2.1.6, 2.1.11, 2.1.24 in the collection of tasks [7] can be used as tasks contributing to the development of the ability to curtail mathematical reasoning.

VI. Ability flexibility of mathematical thinking – the ability to purposefully change actions when the conditions of a task change, as well as easily switch from one solution method to another.
Task. When transferring the point in parallel $(1;1)$ moves to a point $(-1;0)$. Which point does the origin go to?
Solution. Any parallel transfer is given by the formulas

$$x' = x + a,\ y' = y + b.$$

The student will find this formula thanks to mathematical memory and, according to the condition, will use it to find the offset point of the origin due to the flexibility of mathematical thinking.
Since the point is $(1;1)$ moves to a point $(-1;0)$, that $-1 = 1 + a,\ 0 = 1 + b$. From here $a = -2,\ b = -1$. Thus, our parallel transfer, which translates the point $(1;1)$ in $(-1;0)$, it is defined by formulas $x' = x - 2,\ y' = y - 1$. Substituting the coordinates of the origin into these formulas $(x = 0,\ y = 0)$, we get $x' = -2,\ y' = -1$. So, the origin goes to a point $(-2;-1)$.
Tasks 2.3.28, 2.3.29, 2.3.31 from the collection of tasks [8] can be used as tasks for developing the flexibility of mathematical thinking of students.

VII. The ability of rational mathematical thinking - the student's ability to appropriately choose a solution method (reasoning) that leads to the answer of the problem at the lowest cost.
To develop the ability of rational mathematical thinking, it is advisable to use the following tasks.
Task. Let two semi-straight lines a and b, which do not belong to the same straight line, originate from the point O. Then, if the semidirect h, starting from the point O, intersects the segment AB with the ends on the semidirect a and b, then it intersects any other segment with the ends on these semidirect.

Solution. It is clear that, by the condition of the problems, the segments AB and XY, the semidirect a and h are in one of the half-planes defined by the straight line containing b, and the complement h' of the semidirect h to the straight line (we denote it) it is located in another half-plane. Applying problem 3 to triangles ABX and BXY and the straight line c sequentially, we conclude that it intersects BX and YX.

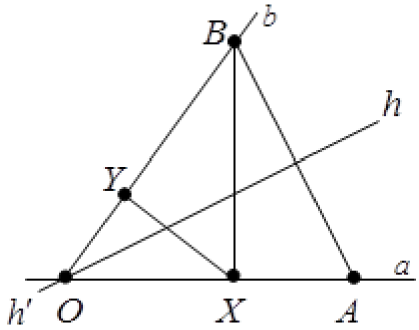

Fig.5.

Since h' and XY If they are in different half-planes and, therefore, do not intersect, it follows that h intersects (Fig. 5)

This task helps students develop rational mathematical thinking.

Tasks 2.2.7, 2.2.33, 2.2.38 from the collection of tasks [8] can be used as tasks for developing students' ability to think rationally.

The use of this formula is proof of the student's creative ability.

Tasks 2.3.67, 2.2.74 from the collection of tasks [8] can be used as tasks for developing students' creative thinking abilities.

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