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TOPOLOGICAL PROPERTIES OF THE SORGENFREY LINE

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Abstract

This article investigates the topological properties of the Sorgenfrey line, which is considered one of the fundamental examples in general topology. The Sorgenfrey line is defined as the set of real numbers endowed with a topology generated by the family of half-open intervals [a,b)[a,b][a,b]. The paper analyzes its main properties, including the fact that it is a Hausdorff space and first countable, but not second countable. In addition, it is shown that the Sorgenfrey line is neither separable nor Lindelöf. Special attention is given to its normality and to problems related to its Cartesian product, in particular the non-normality of the Sorgenfrey plane. The obtained results highlight the role of the Sorgenfrey line as a classical counterexample in topology and contribute to a deeper understanding of the relationships between various topological properties.

Keywords: Sorgenfrey line, topological space, Hausdorff space, first countable space, second non-countable, separability, Lindelof property, normal space, Sorgenfrey plane, counterexample.

Annotatsiya:

Ushbu maqolada umumiy topologiyada muhim misollardan biri hisoblangan Zorgenfrey chizig‘ining topologik xossalari batafsil o‘rganiladi. Zorgenfrey chizig‘i haqiqiy sonlar to‘plamida yarim ochiq [a,b)[a,b)[a,b) intervallar oilasi yordamida hosil qilingan topologiya sifatida ta’riflanadi. Maqolada ushbu fazoning asosiy xossalari, jumladan, Hausdorff fazo ekanligi, birinchi hisobli bo‘lishi, ammo ikkinchi hisobli emasligi, separabel va Lindelyof xossalariga ega emasligi isbot va tahlillar asosida ko‘rib chiqiladi. Shuningdek, Zorgenfrey chizig‘ining normal fazo ekanligi hamda uning Dekart ko‘paytmasi bilan bog‘liq masalalar, xususan Zorgenfrey tekisligining normal emasligi muhokama qilinadi. Olingan natijalar Zorgenfrey chizig‘ining klassik qarshi misol sifatidagi o‘rnini yoritib berib, topologik xossalar o‘rtasidagi nozik farqlarni anglashga xizmat qiladi.

Аннотация (Russian):

В данной статье исследуются топологические свойства прямой Зоргенфрея, являющейся одним из классических примеров в общей топологии. Прямая Зоргенфрея определяется как множество действительных чисел, наделённое топологией, порождённой семейством полуоткрытых интервалов вида [a,b)[a, b)[a,b). В работе рассматриваются основные свойства этого пространства, в частности, то, что оно является хаусдорфовым и первосчётным, но не второсчётным. Показано также, что прямая Зоргенфрея не является сепарабельной и не обладает свойством Линделёфа. Особое внимание уделяется нормальности данного пространства и вопросам, связанным с его декартовым произведением, в частности, ненормальности плоскости Зоргенфрея. Полученные результаты подчёркивают значение прямой Зоргенфрея как классического контрпримера и способствуют более глубокому пониманию взаимосвязей между различными топологическими свойствами.

the set of real numbers $\{(a;b]: a,b \in R, a<b\}$ is called the Sorgenfrey line (or left arrow) and $S_{\leftarrow}$ is denoted by . $S_{\leftarrow}$ We call it the left arrow of the topological space and τ_0 denote the topology defined by or . $\tau_{\leftarrow}$

in the set of real numbers $\{[a;b)\ a,b \in R, a<b\}$ is called the Sorgenfrey line (or right arrow) and $S_{\rightarrow}$ is denoted by . $S_{\rightarrow}$ We call it the right arrow of the topological space and τ_S denote the topology defined by or . $\tau_{\rightarrow}$

Let us assume that X,Y the sets satisfy the following conditions $X,Y \subset R$, and let us denote by the symbol $Y \subset X$. X_Y a topological space in which x the neighborhood basis of a point B_x is defined by the condition:

$B_x = \{(x-\varepsilon;x] \cap X : \varepsilon > 0\}$ if $x \in X \setminus Y$,

$B_x = \{[x;x+\varepsilon) \cap X : \varepsilon > 0\}$ if $x \in Y$.

If $X = S$ and $Y = A$, then S_A we form a space. In this A case, the set is given by the "right arrow" topology. S_A We call this space a modification of the Sorgenfrey line and τ_A denote the topology on this space as . S_A The properties of the space are similar to the Sorgenfrey properties.

$A \subset R$ Let the set x be the following neighborhood of the point

$B_x = \{(x-\varepsilon;x] : \varepsilon > 0\}$ if $x \in R \setminus A$

$B_x = \{(x-\varepsilon;x+\varepsilon) : \varepsilon > 0\}$ if $x \in A$

If we introduce a basis, $H(A)$ we denote the topological space by . $H(A)$ The space is called the Hattori space and is expressed by the topology of this space . τ_A A space $(R,\tau(A))$ that has the Hattori topology is called the Hattori space and R is denoted by . This space and some of its properties were first shown in the works of Y. Hattori. In particular, the Hattori space for is regular, the generatrix Lindelof (hence normal), the generatrix separable, and the Beir space. Note that $A \subset R$ for any $A \subset R$ $\tau_E = \tau(R) \subset \tau(A) \subset \tau_A$

In particular, $S_{\varnothing} = S = H(\varnothing)$.

1. Confirmation. Every What $A \subset R$ part of the set S_A is the space completely imperfect?

Proof. Suppose that $K \subset S_A$ the compact set is not countable. In that case, either one $K \cap A$ of its $K \setminus A$ sets is uncountable. Suppose that $K \cap A$ the set is uncountable.

an interval is found $\varepsilon_k > 0$ for $(k-\varepsilon_k, k)$ each point, and it is reasonable $k \in K \setminus A$ for $(k-\varepsilon_k, k) \cap K = \varnothing$. Indeed, if such intervals do not exist, then R we k consider an increasing sequence $U_n - k_n$ for , $\{k_n \in K : n \in N\}$ which approaches the point in this case. $i \neq j$, $k_i \neq k_j$ Let be the neighborhood of the point, then it $U_n \cap K = \{k_n\}$ will be. It follows that $\{(-\infty;k_n) : n \in N\} \cup \{[k;+\infty)\}$ -

K a finite part of the covering that contradicts the coverage of cannot be separated from the covering. Thus, $(k-\varepsilon_k;k)$ the interval in the view exists and $(k-\varepsilon_k;k)\cap K=\varnothing$ is reasonable.

Obviously,

$$\left(k^{'}-\varepsilon_{k^{'}};k^{'}\right)\cap\left(k^{"}-\varepsilon_{k^{"}};k^{"}\right)=\varnothing$$

$k^{'}\neq k^{"}$ for , here $k^{'},k^{"}\in K$. F a r a zg a kor a , bizd a $(k-\varepsilon_k;k)$ s is a non - intersecting interval , here $k\in K$ This is not possible because there is a white circle S_A around the waist .

$K\setminus A$ The collection is in perfect condition , so it can be proved .

2. T a sdiq. Each part is a Beir phase for S_A the set . $A\subset R$

The outer part of the collection is also the densest part of the Beir phase [page 5,228] . G_δ

1. Result. The following statements are valid:

1) For any $A\subset R$ subset of v, the space $T\subset S_A$ for a dense subset of v (T,τ_A) everywhere G_δ is a Beir space. In particular, (J,τ_A) the set of irrational points is a Beir space.

2) $G-S_A$ If a space has an open subset, (G,τ_A) the space is a Beir space.

3. Proof. For S_A any subset of a space, $A\subset R$ it is a generational Beir space.

Proof. S_A Since the space is regular and satisfies the first axiom of countability, to prove that it is a descendant Beir space, S_A it suffices to show that the space does not have a countable closed subspace without isolated points. Let F the set S_A have no closed and isolated points. In the Euclidean topology of the straight line , F the closure of the set $\overline{F}$ does not include isolated points. According to [1], F the set is uncountable. $\overline{F}\setminus F$ The reason for being countable is that F the set is uncountable.

$f:K\to D$ reflection K –Sorgenfrey $D=\{0,1\}$ reflects the straight line on a two-point graph using the following formula:

$$f(x)=\begin{cases}0, agar\ x<0\\ 1, agar\ x\geq 0\end{cases}$$

This reflection will be open and closed. $\tau_S=\{[a;b,a\in R,b\in R,a<b)\}$ –The family of open sets of the Sorgenfrey line. Hence

$(-\infty;0)=\sum_{n=1}^{\infty}\left[-n;\frac{1}{n}\right)$ will be, $\left[-n;\frac{1}{n}\right)$ – τ_S because it is open $f\left(\left[-n;\frac{1}{n}\right)\right)=0$ - D it is open and $f([0;\infty))=1$ - it is open D, so f –is open reflection. $(-\infty;0)=X\setminus[0;\infty)$ - because it is closed and $[0;\infty)=X\setminus\sum_{n=1}^{\infty}\left[-n;\frac{1}{n}\right)$ closed, it is $f((-\infty;0))=0-D$ also closed and $f([0;\infty))=1-D$ closed, so $f-$ it is closed reflection. From this $f-$ it follows that reflection is open-closed.

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