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UNSTATIONARY FLOW OF A VISCOELASTIC FLUID IN A PLANE CHANNEL

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Abstract
The article formulates a generalized model of an elastic-viscous fluid, in particular, from this model one can obtain Newtonian, generalized Newtonian, Maxwell and other models. Basically, the generalized model of a viscoelastic fluid is built on the basis of the topological hypothesis of Astarte and Mariucci and the axiomatic principles of Truesdell and Knoll. The developed generalized model of a viscoelastic fluid is convenient for solving engineering problems and thus is easily implemented for studying the flow of non-Newtonian fluids in a flat channel and in a circular cylindrical tube.
Keywords: Elastic, viscous, Newton's model, Maxwell's model, axiom, flat channel, cylindrical tube.

Introduction
Studies [3-5] show that solving the problem of unsteady flow of a viscoelastic fluid in pipelines leads to significant mathematical difficulties. Therefore, when solving problems, methods of simplification are used, or the problem [7,8] is solved in a one-dimensional formulation with lumped velocities over the pipe section. In the majority of works [3-5], it is argued that transient processes occur during unsteady flow, which significantly depend on the properties of the fluids flowing in them. In this article, specific problems are solved about the unsteady flow of an elastic-viscous fluid in a flat channel and transient processes are analyzed. In addition, when solving one-dimensional problems, the data obtained are not only of great importance for the detection of new hydrodynamic effects, but also are a reliable source for comparison with the results of studying an elastic-viscous fluid in a more complex formulation. On the basis of the rheological models of an elastic-viscous fluid proposed in [4,5], we will solve non-stationary problems in pipes and channels, where the fluid is assumed to be elastic-viscous and incompressible, and its motion is laminar and axisymmetric.

Main part
In this case, the movements of the liquid in the pipes, taking into account its rheological properties, are described by simplified equations of the following form

$$\begin{cases} \rho \dfrac{\partial u}{\partial t} = -\dfrac{\partial p}{\partial x} + \dfrac{\partial}{\partial y}\left(\tau_{xy}\right), \quad \dfrac{\partial p}{\partial y} = 0, \\ \tau_{xy} = \sum\limits_{k=1}^{\infty} \tau_{k,xy}, \quad \dfrac{\partial \tau_{k,xy}}{\partial t} + \dfrac{g_k}{\lambda_k}\tau_{k,xy} = p_k \dfrac{\partial u}{\partial t}, \\ \dfrac{\partial p_k}{\partial t} + \dfrac{g_k}{\lambda_k} p_k = \dfrac{\eta_k}{\lambda^2{}_k} f_k . \end{cases} \quad (1)$$

To solve the system of equations (1), it is necessary to formulate the initial and boundary conditions. We assume that at, the liquid in the initial state is assumed to be "at rest", i.e.

$$u = 0, \quad \frac{\partial p}{\partial x} = 0 \quad \text{at} \quad t = 0 \qquad (2)$$

$$u = 0, \ at \ y = h, \ \frac{\partial u}{\partial y} = 0 \ at \ y = 0 \ u \neq 0, \ \frac{\partial p}{\partial x} \neq 0 \quad at \quad t \succ 0 \qquad (3)$$

Boundary value problems for equations (1) with boundary and initial conditions (2) and (3) can be solved in the final form only for flows, when $f_k = 1, \quad g_k = 1$in equation (1), and for a fluid with a relaxation time spectrum

$$\lambda_k = \frac{\lambda}{k^{\alpha}}, \qquad \eta_k = \frac{\eta}{\xi(\alpha) k^{\alpha}}.$$

The most complex fluid flows with a nonlinear spectrum of relaxation times require the use of a cumbersome apparatus of mathematical physics or a numerical method. Linearized equations (1) and initial and boundary conditions (2), (3) using the Laplace-Carson transformation [6], in time, taking into account the initial conditions, can be written as:

$$\begin{cases} \dfrac{\partial \bar{\tau}_{xy}}{\partial y} - \rho s \bar{u} = \dfrac{\partial \bar{p}}{\partial x}, \\ \dfrac{\partial \bar{p}}{\partial y} = 0, \quad \bar{\tau}_{xy} = \sum\limits_{k=1}^{N} \bar{\tau}_{k,xy}; \\ s \bar{\tau}_{k,xy} + \dfrac{1}{\lambda_k} \bar{\tau}_{k,xy} = \dfrac{\eta_k}{\lambda_k} \dfrac{\partial \bar{u}}{\partial y} \end{cases} \qquad (4)$$

From equation (4) one can find $\bar{\tau}_{rx}$:

$$\bar{\tau}_{xy} = \sum_{k=1}^{N} \frac{\eta_k}{1 + s\lambda_k} \frac{\partial \bar{u}}{\partial y} = \eta(s) \frac{\partial \bar{u}}{\partial y}, \qquad (5)$$

Where

$$\sum_{k=1}^{N} \frac{\eta_k}{1 + s\lambda_k} = \eta(s)$$

Substituting the obtained expression (5) into equation (4), we obtain

$$\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\rho_0 s}{\eta(s)} \bar{u} = \frac{1}{\eta(s)} \frac{\partial \bar{p}}{\partial x}. \qquad (6)$$

In equation (6) $\frac{\partial \bar{p}}{\partial x}$ does not depend on the channel width, so its solution can be obtained in the form of a trigonometric function. It is believed that the variable x is a "frozen" parameter in the equation:

$$\bar{u} = \frac{1}{\rho\, s}\left(-\frac{\partial \bar{p}}{\partial x}\right)\left(1 - \frac{\cos\left(i\sqrt{\frac{\rho_0 s}{\eta(s)}}\, y\right)}{\cos\left(i\sqrt{\frac{\rho_0 s}{\eta(s)}}\, h\right)}\right). \qquad (7)$$

This expression only has simple poles

$$s = 0, \quad s = -\nu \frac{\bar{s}_{ni}}{h^2}$$

where $\bar{s}_{ni}$ is the root of the transcendental equation

$$\bar{s} + \bar{\eta}(\bar{s}) \ \ (\bar{s}) \frac{(2p+1)^2}{2} \pi^2 = 0: \qquad (8)$$

Here

$$\bar{\eta}(\bar{s}) = \frac{1}{\xi(\alpha)} \sum_{k=1}^{\infty} \frac{1}{k^\alpha - EL\bar{s}}. \qquad (9)$$

is the Shulman-Husid rheological equation, where $EL = \frac{\nu\lambda}{h^2}$. It can be solved $\bar{\eta}(\bar{s}) = \frac{1}{\xi(\alpha)} \sum_{k=1}^{\infty} \frac{1}{k^\alpha - EL\bar{s}} \approx \frac{1}{\xi(\alpha)} \sum_{k=1}^{\infty} \frac{1}{k^\alpha} = 1.$ Hence, $\eta(s) = \eta\bar{\eta}(\bar{s}) = \eta$ in this case, all the results obtained for the Newtonian fluid are true for an elastic-viscous fluid when the rheological equations are given in the form of the Shulman-Husid model.

Conclusion

In the second case, when $|\lambda s| \square 1$ (8) is the transcendental equation when the expression $\bar{\eta}(\bar{s})$ is replaced by its asymptotic expression $\bar{\eta}(\bar{s}) = \frac{\pi}{\xi(\alpha)\alpha \sin(\pi/\alpha)(\lambda s)^{(1-1/\alpha)}}$

Then (8) at $\alpha = 2$ has the form

$$\bar{s} + \frac{\pi}{2\xi(2)(-EL\bar{s})^{\frac{1}{2}}} \frac{(2p+1)^2}{2} \pi^2 = 0:$$

or $\bar{s}_{in} = \frac{\pi^2}{4\sqrt[3]{\xi^2(2)EL}}(2n+1)^{4/3}(\frac{1}{2}+i\frac{\sqrt{3}}{2})$

The final solution would be like this

$$\frac{u(0,t)}{u_{0max}} = \left[1-32\sum_{n=0}^{\infty}(-1)^n \frac{\sqrt[3]{\xi^2(2)EL}}{3(2n+1)^2\pi^3\sqrt[3]{2n+1}} e^{-\frac{\nu}{h^2}\bar{s}_n t}\right]$$

Using the obtained solution, numerical calculations can be made for the Shulman-Husid model.

References

1) Navruzov K., Xakberdiev J.B. Dinamika nenyutonovskix jidkosti. Tashkent, “Fan”, 2000.-246 s.
2) Navruzov K. Pulsiruyushee techenie uprugovyazkoy jidkosti v ploskoy trube// Uzb.Jurn. «Problemiy mexaniki», 2002, №1. – S.
3) Shulman Z.P. Nestatsionarniye protsessiy reodinomiki i teplomassoobmena. – Minsk,1983. –169 s.
4) Shulman Z.P., Xusid B.M. Nestatsionarniye protsessiy konvektivnogo perenosa v nasledstvenniyx sredax. – Minsk, 1983. – 256 s.
5) Shulman Z.P., Aleynikov S.M., Xusid B.M. Perexodniye protsessiy pri sdvigiyx techeniyax vyazkouprugoy jidkosti.1. Rasprostranenie sdvigovoy volniy. // IFJ, 1982, t. 42, №6. – S. 992-1000.
6) Slezkin N.A. Dinamika vyazkoy nesjimaemoy jidkosti. – M.: Gostexizdat, 1956. – 520 s.
7) Fayzullayev D.F., Navruzov K. Gidrodinamika pulsiruyushix potokov, –Tashkent, “Fan”,1986. – 192s.
8) Navruzov K. Biomexanika krupniyx krovenosniyx sosudov. –Tashkent, “Fan va texnologiya”, 2011. – 144 s.