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A REVIEW ON IMAGE REGISTRATION TECHNIQUES FOR BLUR REMOVAL

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Abstract— **The original method utilizes the phase of the images and has its roots on phase correlation (PC) registration. The even powers of the normalized Fourier transform of an image are invariant to centrally symmetric blur, such as motion or out-of-focus blur. Then use these results to propose blur-invariant phase correlation. The method has been compared to PC registration with excellent results. The method works for unknown blurs assuming the blurring PSF exhibits an *N*-fold rotational symmetry. It does not require any landmarks. We explicitly address only registration with respect to translation, but the method can be readily generalized to rotation and scaling. Now we also generalize the theory to the case of dihedrally symmetric blurs, which are produced by the PSFs having both rotational and axial symmetries. Such kind of blurs is often found in unfocused images acquired by digital cameras, as in out-of-focus shots the PSF typically mimics the shape of the shutter aperture. This makes our registration algorithm particularly well-suited in applications where blurred image registration must be used as a preprocess step of an image fusion algorithm, and where common registration methods fail, due to the amount of blur. We demonstrate that the proposed method leads to an improvement of the registration performance, and we show its applicability to real images by providing successful examples of blurred image registration followed by depth-of-field extension and multichannel blind deconvolution.**

Keywords— Image registration, blurred images, N-fold rotational symmetry, dihedral symmetry, phase correlation.

I. INTRODUCTION

Image registration, a process of spatial overlaying two or more images of the same scene, is one of the most important tasks in image pre-processing. As such, it has received considerable attention and hundreds of papers have been published on this subject. Since there obviously does not exist any unique always-optimal approach, many authors have proposed specific registration methods for particular application areas, for particular kind of data and/or for a certain class of between-image deformations. In many cases, the images to be registered are blurred. The blur may originate from camera shake and/or wrong focus, scene motion, atmospheric turbulence, sensor imperfection, low sampling density and other factors. The demand for blurred image Registration comes namely from applications, where a short sequence of low-quality images is used to produce a single sharp high-resolution output. Registration of blurred images requires special methods because general registration methods usually do not perform well on blurred images. The out-of-focus PSF's of several common cameras has shown that they not only have N-fold rotational

symmetry but they mostly also have axial symmetry with respect to N axes; such "combined" symmetry is in mathematics called dihedral symmetry. We can see the PSF's of three cameras obtained by taking a photo of a single bright point. The shape of the PSF is sometimes apparent even in real scenes. Axial symmetry was not considered at all, although it carries additional information about the PSF. In this system, we extend the theory and the registration method originally proposed to the blurs with dihedral symmetry by defining new dihedral projection operators. We show this extension has a practical impact because we essentially utilize more information about the PSF compared to the pure N-fold rotational assumption, which increases the registration performance particularly in case of noisy images and of a low image overlap.

II.BLUR-INVARIANT PHASE CORRELATION
A. *Phase Correlation Method*

The PC image registration method .The method is based on the Fourier shift theorem, which states that if two images f1 and f2 and differ only by displacement(X_0, Y0)

$$f_2(x,y) = f_1(x - x_0, y - y_0)$$

(1)

their Fourier transformsF1 and F2 and are related by

$$F_2(u,v) = F_1(u,v)e^{-i(ux_0+vy_0)},$$

(2)

This means that the images f1andf2 have the same Fourier magnitude, while the phase difference is directly related to their spatial displacement. As the Fourier transform $F(u,v)$ itself is a complex function and can be written by its magnitude and argument, namely

$$F(u,v) = |F(u,v)| \, e^{-i\phi(u,v)}$$

(3)

It turns out that the normalized cross power spectrum of the two images defined as

$$S(u,v) = \frac{F_2(u,v)F_1^*(u,v)}{|F_2(u,v)F_1^*(u,v)|} = e^{-i(ux_0+vy_0)}$$

(4)

Where* denotes complex conjugate, has the phase corresponding to the phase difference of the images f1and f2.

B. *Blur-Invariant Phase Correlation*

In this section, the theory of BIPC is derived. The derivation has similarities with the derivation of the frequency domain BIFs. If noise is neglected, can be expressed in the Fourier domain using the convolution theorem by

$$G(u, v) = F(u, v) \cdot H(u, v) \qquad (5)$$

and in the phasor form by

$$G(u, v) = |G(u, v)| \, e^{-i\phi_g(u,v)} \qquad (6)$$

If the Fourier transform $G(u, v)$ is normalized by its magnitude, only the complex exponential containing the phase remains namely

$$\frac{G(u, v)}{|G(u, v)|} = e^{-i\phi_g(u,v)} = e^{-i(\phi_f(u,v)+\phi_h(u,v))} \qquad (7)$$

Where $\Phi_f(u, v)$ is the phase of the original image and the phase of the blur PSF. we registered pairs of blurred, noisy, and translated images of size 300 300 taken from either 400 image in Fig. 1. Each image in a pair was motion blurred in a different and random direction, and the other image was corrupted by additive Gaussian noise resulting in a peak signal-to noise ratio (PSNR) of 34 dB. For subpixel translation we had to perform interpolation. Translations were randomly generated in the range [50, 50]. Fig. 2 summarizes the results when the blur length is increased in steps of one, and the experiment is repeated a thousand times for each blur length and for each of the images in Fig. 1.The root mean square (RMS) registration error (in pixels) in the case of PC increases as the length of blur increases. On the contrary, BIPC seems to perform nearly equally despite blur, and we really achieve subpixel registration accuracy for even heavily blurred images
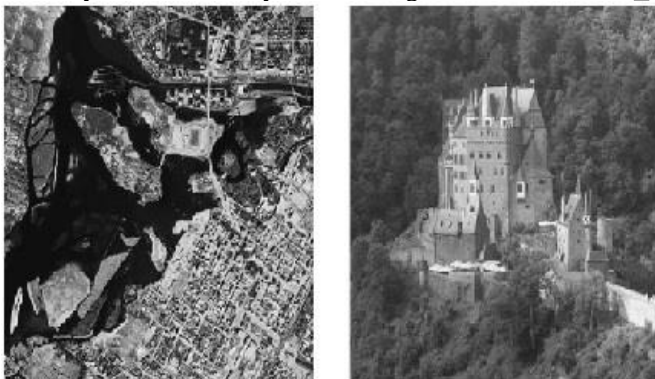

Fig. 1. Images used in the experiments.

We used a high-resolution (9 Mpix) version of the left image in Fig. 1 and performed the translation in integer pixel accuracy after which we down sampled the image resulting in subpixel shifts. In this way, we avoided the subpixel interpolation, which would create additional blur. Fig.2 shows the resulting RMS registration error when the blur length is in the range [0, 1] and the experiment is repeated a thousand times for each blur length in steps of 0.1. The curves for both PC and BIPC are shown for three different noise levels. As expected, when noise is added, the RMS registration error for each method increases. Without blurring, the RMSE in the case of PC is around 0.05 pixels and in the case of BIPC 0.1 pixels. The limit after which it is better to use BIPC instead of PC seems to be between the

blur lengths of 0.5 and 0.9. In practice, one should always use BIPC if there is a possibility of some motion blurs existing in the images. In the presence of noise, ordinary PC seems to be more robust for very large translations, i.e. in the case of only a small overlap, provided that blurring is subtle. When the PSNR is less than 28 dB, BIPC often fails if the translations are larger than a quarter of the image size.

II. N-FOLD BLUR-INVARIANT PHASE CORRELATION

A. Phase Correlation of Primordial Images

To apply standard phase correlation to primordial images and in this way to obtain a blur-invariant registration method. However, the cross-power spectrum

$$C(\mathbf{u}) = \frac{I_N^{(f)} I_N^{(g)*}}{\left| I_N^{(f)} \right| \left| I_N^{(g)} \right|} \qquad (8)$$

produces neither a single peak, nor any other easy-to-detect pattern in $F^{-1}\{C\}(\mathbf{x})$. The main reasons are that, unlike "conventional" image spectra, the magnitude $|I_N|$ is not preserved if f is shifted and also that projection operators do not commute with a shift. The relation between $C(\mathbf{u})$ and Δ can still be derived and (at least theoretically) used for estimating the registration parameters. However, this process, when implemented numerically, is not robust. Its sensitivity to non-complete overlap of the images and to noise makes it unreliable.

B. Phase Correlation Between Separated N-Fold Invariants

In the previous Section, we have discussed the problems arising from the approach of estimating the shift between two blurred images f and g using the strategy of computing phase correlation of their primordial images. We now present an alternative method whose properties will enable us to obtain robust estimate of the translational shift between f and g. We start by observing that the invariant expressed is not the only possible formulation of an N-fold blur invariant. Let's introduce the following operators:

$$K_j^{(f)}(\mathbf{u}) = \frac{F(\mathbf{u})}{F(\mathbf{R}_j\mathbf{u})} \quad j = 1, \ldots, N \qquad (9)$$

C. Fitting a Circle

Detecting the peak in each $F^{-1}\{Cj\}$ is simple just by identifying the maximum value, so we find a peak location $\mathbf{P}_j$ such that $F^{-1}\{Cj\}(\mathbf{P}_j) > F^{-1}\{Cj\}(\mathbf{x})$, for all other $\mathbf{x} \in Z^2$. Theoretically, it should be the only non-zero value there. In practice, this is not the case due to finite precision, but still the peak significantly exceeds the other values in $F^{-1}\{Cj\}$ and is easy to locate. After detecting all the peaks $\mathbf{p}_1, \mathbf{p}_2 \ldots \mathbf{p}_{N-1}$ a crucial step is to fit the circle, the center of which determines the shift parameters. Note that always $P_N = \mathbf{0}$ and hence we constrain the circle to pass through the origin in any case. Each pair was registered (which means here that the between-patch shift was estimated) by the new method

described in Section III. In this simulated experiment, the blur has almost circular symmetry, so N should be chosen "sufficiently large" (the theoretical value is $N = \infty$, actual ground truth value is $N = 32$, but we set $N = 8$ in this experiment to avoid a perfect match which is not realistic in practice). A larger N would provide slightly better results but at the expense of computing complexity, while choosing a smaller N would decrease the performance slightly. For comparison, traditional phase correlation was applied as well

Fig. 2. The original image (top left) and eight noisy instances with the amounts of noise used in the experiment

TABLE I
Robustness of the *n*-fold phase correlation, $n = 8$,
With respect to additive gaussian noise. overlap
Of the patches 70%, blur radius 7 pixels

Gaussian Noise σ	5	10	15	20	25	30	35	40
Misregistrations (out of 100)	0	5	9	11	19	31	38	43

On the other hand, the proposed *N*-fold blur invariant phase correlation is designed to address this drawback, and in fact, it did not yield any misregistration for overlaps larger than 50% and only a few for 50%, which is a perfect performance. If the overlap is only 40%, some of the peaks which are fitted by a circle fall beyond the support we are working on and, due to the periodicity of DFT, they appear in locations which are "modulo the image size". This happens particularly to the correlation peaks extracted from C_j using (6), when j is close to $N/2$. Although the $L0.2$ fit is robust to outliers, it fails if such points are too many. In our experiment the mean misregistration rate in case of 40% overlap is 6 out of 30 trials (this rate is nearly independent on the blur amount) which is still much better than that of traditional phase correlation. The median error and the median absolute deviation obtained with the *N*-fold phase correlation were remarkably all zero for all combinations of overlap/blur radius. This essentially means that the registrations yielded either an error of 0 pixel, or a random

shift, but the random shifts are treated as outliers by these robust statistics. An additional experiment was carried out in order to test the robustness of the proposed method with respect to noise. In this experiment, 100 patch pairs were randomly selected as described above. The overlap was kept fixed at 70%, and the blur radius was set to 7 pixels. The blurred images were degraded by different levels of additive white Gaussian noise. Since the pixel intensities of the images lie within the range [0...255], the noise standard deviations used were 5, 10, 15, 20, 25, 30, 35, and 40. Fig. 5 depicts the effect of the noise levels used in the experiment. The proposed method was utilized to align the sharp images with the blurred and noisy versions. The results, demonstrating reasonable robustness of the method, are shown in Table I. We can see that the method yields almost perfect results for $N = 4$ and tolerates a wrong choice of the fold number in certain cases. Let us denote the chosen fold number as M. This tolerance depends mainly on the distance between the spectral peaks produced by the correct N and those by M. Note, that the "width" of the M-peaks is inversely proportional to the distance from the nearest N-peak.

III. N-FOLD DIHEDRAL BLUR
A. *Blur Invariant Operators*

We have seen that the construction of the blur-invariant registration method is possible thanks to the N-fold symmetry of the PSF. Any extension and generalization to other types of PSF's must be based on studying their symmetric properties, which consequently should enable to find proper projection operators and proceed analogously. Symmetry in 2D has been traditionally studied in group theory. It is well known that in 2D there exist only two kinds of symmetry groups which are relevant to our problem: cyclic groups C_N that contain N-fold rotational symmetry and dihedral groups D_N that contain rotation and reflection symmetry. The relationship between these two symmetries is that if a function shows N-fold rotational symmetry, then it may only have either none or N symmetry axes. On the other hand, having N symmetry axes immediately implies N-fold rotational symmetry. Therefore, $D_N \subsetneq C_N$ for finite N, and $D_\infty = C_\infty$. Hence, it is meaningful to deal with registration of images with PSF's having dihedral symmetry, for two reasons – such situation appears frequently in practice, and at the same time is mathematically tractable. Since the group C_N has only one generator (elementary rotation R_1), while D_N has two generators (elementary rotation and reflection), we may expect that in case of dihedral symmetric PSF there exist two times more projections analogous and the method will double the number of delta-peaks, which should further lead to a more robust fit, especially in such cases when the localization of the peaks is difficult due to noise.The reflection operator S is given as where α is the angle

between the reflection line (symmetry axis) and the horizontal axis.

$$S(\mathbf{x}) = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(10)

B. Image Registration Algorithm

The invariants introduced in the previous portions will be now used to design a robust blur-invariant registration method. The main idea is that we may consider the invariants to be Fourier transforms of hypothetical non-blurred images, which can be registered by phase correlation. The registration is completed by fitting a circle over all the delta-peaks. The fitting algorithm minimizes the p error and is exactly the same as that including the choice of an appropriate p. The only difference is that here we fit a double number of peaks (2N instead of N), which generally leads to an improvement, since we experimentally observed that the probability distribution of localization error is the same as the one we reported for all peaks, both N-fold and dihedral peaks. From this it is clear that adding new peaks cannot worsen the fit. This is the main advantage of the presented method. We calculate the normalized cross-power spectra

$$C_j = \frac{\mathcal{K}_j^{(f)} \mathcal{K}_j^{(g)*}}{\left| \mathcal{K}_j^{(f)} \mathcal{K}_j^{(g)} \right|}$$

(11)

and

$$B_j = \frac{\mathcal{L}_j^{(f)} \mathcal{L}_j^{(g)*}}{\left| \mathcal{L}_j^{(f)} \mathcal{L}_j^{(g)} \right|}$$

(12)

C. Estimation of the Symmetry Axis

While N is fixed for the particular camera and mostly known in advance, the orientation of the symmetry axis of the PSF depends on the camera rotation and also changes as the aperture opens/closes, so it may not be known a priori. In some cases the axis orientation can be estimated directly from the bright patterns in the blurred image, but in other cases a general estimation algorithm is required. We propose here a simple algorithm to estimate the orientation of one of the symmetry axes of a PSF having dihedral symmetry.

D. Estimation of N

The parameter N of the blur PSF is generally determined by the mechanical design of the shutter, and it normally coincides specific device. N is often known in advance, or at least, easy to obtain by visually inspecting either the shutter, or a blurred image where the shape of the PSF manifests itself. However, in some cases, the user might need to estimate N directly from the blurred image. The main difference in performance of both methods appears when registering noisy images with low overlap, as is demonstrated by the following experiment. We took ten 4-megapixels images with a consumer camera and converted them to grayscale for simplicity (the algorithm works with color images as well, treating them band by band). For each of these, a corrupted version was artificially created by blurring the original image with a PSF of 31×31 pixels, a specified degree N of dihedral symmetry, and random but known symmetry axis. In addition to the blur, each image was corrupted by additive Gaussian white noise. The image intensity values were in the range [0, 255], and the noise standard deviations σ used in the experiments were 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100, respectively. From each of these images, ten pairs of square patches of the size 256×256 pixels and a percentage of 50% of overlapping area were extracted at random locations, one from the sharp image, and the other one from its blurred and noisy version, giving a total number of 1100 image pairs to be registered (100 pairs on each noise level). The new dihedral phase correlation and the N-fold phase correlation were used to register them. We evaluated the performance of the methods by counting the number of misregistrations, where any registration error greater than 1 pixel in terms of Euclidean distance is considered as a misregistration. The average increase in performance observed using the proposed method was approximately 11%. Although the improvement is observable on each noise level (including the noise-free case), it increases as the noise variance increases. When using the algorithm for symmetry axis estimation, the average increase in performance was 8% . When we did not apply any axis estimation method and "estimated" the axis orientation randomly, the method performance converged to that of the N-fold method. Our algorithm for the circle-fit can easily tolerate slightly more than 50% of outliers.

IV. EXPERIMENTS

The performance of BIPC was compared to the BIFs. In our experiments we used the spatial version of the BIFs of order seven. The spatial BIFs exhibit similar invariance to blurring as BIPC, but they are far slower to compute. So, the aim was to show that BIPC can perform similarly with less computation. It is practically impossible to register large images using the BIFs as it would be too slow, and because one image should be included into the second. So, we registered a small template of size T×T to a larger blurred and noisy image of size N×N. We repeated the experiment 100 times by picking template (T=60) from a random sub pixel location of the image (N=200). Before registration, the larger image was blurred by motion blur in a random direction with a blur length of eight and corrupted by additive Gaussian noise resulting in a PSNR of 34 dB. In this experiment, we used real images. The images were captured using a vibrating video camera so that motion blur was created on some of the images. In this case, it is impossible to evaluate which method is better in the subpixel level. So, we compared the BIPC and PC methods using heavily but nearly symmetrically blurred images so

that the errors were large. In Fig.3, a typical registration result is shown obtained using BIPC. The estimated translation is (19.26, 116.85), which seems to be very accurate. For the same image pair, PC estimated the translation to be (13.31, 122.81), which is clearly incorrect. If the blur is strongly non symmetric, also BIPC will fail.

Fig.3. Images contain motion blur and are registered using BIPC. The result seems to be good. For the same image pair PC produced a result that was noticeable incorrect.

See in Fig 4, two pairs of images were taken with a hand-held standard compact camera. The blur was introduced intentionally by changing the focus settings. In each pair, we changed settings between acquisitions and we also moved the camera slightly, which resulted in differently blurred and mutually shifted images. Assuming the PSF's are close to circular, we chose $N = 16$ and applied the *N*-fold phase correlation to register the images. In order to demonstrate one of the possible applications of this registration technique, we used the registered images as an input for the multichannel blind-deconvolution algorithm. In both cases, the resulting deblurred images have much better appearance than the blurred inputs, with almost no artifacts, which is an indication that the registration was accurate enough. The deconvolution algorithm also yields as a by-product the estimated PSF's. One can see they are approximately but not exactly circularly symmetric. The violation of symmetry may originate from second-order errors of the optics and also from estimation errors. In spite of that, the registration algorithm has proven sufficient robustness to such deviations from the assumed PSF shape. We tested our method also in real situations, where both blur and noise

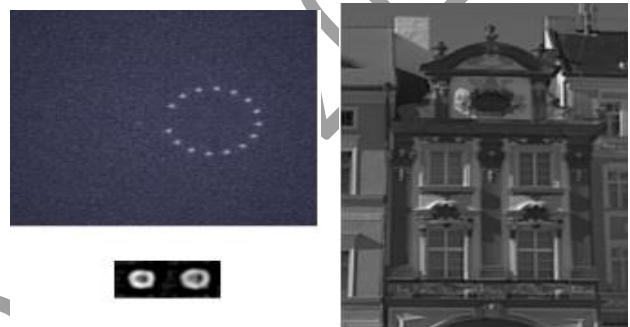*Fig.4.Blurred images to be registered and, the spectral peaks*

Fig.5.The estimated PSF's, and the result of the multichannel deconvolution performed after the registration

were introduced by the camera settings, and the assumed acquisition model was valid only approximately or locally, and the symmetry of the PSF was not perfectly *N*-fold. Here we present the results of an experiment, where we took a short video of a static 3D scene by slowly panning the camera. The depth of field of the camera did not allow both foreground and background to be in focus at the same time. The automatic focus mode of the camera was turned on and the camera always tried to keep the center of the image in focus. Thus the focus settings were automatically re-adjusted when we changed the place we were pointing at. We extracted two frames where the first frame shows the
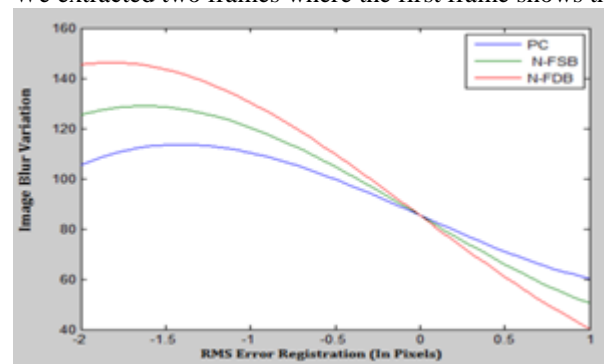

Fig. 6.RMS Error Registration with Image Blur Variation

foreground in focus and the background out of focus, and vice-versa for the second frame. Bright spots in the scene allow us to estimate the symmetry parameters of the PSF.

Clearly, $N = 4$ and one of the symmetry axes is approximately vertical (the estimated angle by means of the algorithm was 4 degrees). As in the previous case, the SIFT-based registration failed due to the blur. Then we used the proposed method of dihedral phase correlation to register these two frames images, and although our method is not specifically designed to handle spatially-varying blur, the result of registration was still accurate. Since we do not know the ground truth, we illustrate the accuracy by performing multifocus fusion. The fused product contains only tiny artifacts that are due to the parallax.

Table 1 shows the comparison of various methods

Title	RMS Error Reg.	Blurred image variation
N-fold Dihedral Blur	1.4	148
N-Fold Symmetric Blur	2.2	126
Phase correlation	3.8	108

V.CONCLUSION

Our results show that registration can be made much more accurately using BIPC than PC when images contain motion blur. As demonstrated, with the subpixel extension of PC, BIPC can produce subpixel registration accuracy even in the case of heavy blur. If it is possible that the images contain motion blur, BIPC should always be used as it performs better compared to PC if the blur length exceeds one pixel. The performance difference for sharp images is negligible, but even a blur of a few pixels can lead to significant error in the case of PC. It works for unknown blurs assuming the PSF's exhibit N-fold rotational symmetry. We proved experimentally its good performance which is not dependent on the amount of blur. It can, of course, be applied to nonblurred images as well, but in that case, it loses its advantage over the standard techniques. It should be noted that there exist (rare) cases of space-invariant blur where our method is not applicable because the PSF has no symmetry, for instance a motion blur along a curved trajectory and high-frequency vibration/shake blur with changing parameters during the acquisition. The method is also not rigorously applicable in the case of space-variant PSF. The implementation of the method is simple and efficient, it consists only of two Fourier transforms, $N − 1$ inverse Fourier transforms, $2N$ rotations and few other simple steps. This could enable its embedded implementation on the camera chips in the future. The new method is based on the assumption that the symmetry axis of the PSF is known. This parameter appears explicitly in the definition of the reflection operator. In certain applications this should not be a serious problem because we can measure the camera settings. For those cases where such assumption might be too restrictive, we proposed a simple algorithm of estimation of the symmetry axis of the PSF. It is demonstrated that the use of the axis estimation algorithm has only a minimal impact in terms of registration accuracy when compared to the ideal case of known PSF axis. scaling are possible in an analogous way to how it was done in traditional phase correlation i.e. by mapping the FT magnitudes into log-polar domain, in which the rotation and scaling are converted to shifts, and rotational symmetry of the PSF is converted to translational symmetry. One could then define projection operators' w.r.t. translational symmetry and proceed analogously to the presented method. However, if the axis orientation is unknown, it must be estimated in advance as in the current version.

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