EXISTENCE OF THE ISOLATED SPECIAL POINTS THREE-DIMENSIONAL DIFFERENTIAL SYSTEMS OF A SPECIAL LOOK

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Abstract
In article the research of a system of the differential equations for a three-dimensional case on distribution of the isolated special points when the right side consists composed from a constant square matrix and a scalar form of degree is considered $k \geq 1$. Here it is supposed that own matrixes valid and various, and are not equal to zero. Linear transformation where the square matrix of transformation is nondegenerate is entered. In a required system the educated new matrix of coefficients has simpler appearance and this matrix can give a type of a Jordan normal form. Further, passing to scalar record of a system, its canonical form is received.
Keywords: Qualitative picture, special point, stability and instability, the first approach, the isolated point, own numbers, scalar form, linear transformation, nondegenerate matrix, Jordan normal form.

Introduction

The foundation of the qualitative theory of the differential equations was laid in classical works of A. Poincare of A.M. Lyapunov. A. Poincare gave rather full qualitative picture of a trajectory of the two-dimensional system containing linear members. In 1928 M. Frommer continued researches in nonanalytic case. Frommer's method gained justification and further development in I.S. Kukless, A.F. Andreyev, Sh.R. Sharipov's works and others.
For a multidimensional system, i.e. in a case $n \geq 3$ was only preliminary results, and the main results about character to a special point of such systems in terms of stability and instability on the first approach are received were generalized by many mathematicians in various directions. In the present to article we will investigate distribution the isolated special systems (1) of points in the assumption that own matrix number A valid and different, and are not equal to zero at $n = 3$.

Main part

Let's consider the systems of the differential equations

$$\frac{dx}{dt} = Ax + xf^k(x), \qquad x = (x_1, x_2, x_3) \qquad (1)$$

where A – a constant square matrix, and $f^k(x)$ – a scalar form of degree $k \geq 1$, $\det \Delta \neq 0$, n=3.

In the present to article we will investigate distribution of the isolated special points of a system (1) in the assumption that own numbers of a matrix A valid and various, and are not equal to zero. Many authors, especially, for a case were engaged in distribution of the isolated special points of a system (1) $x = (x_1, x_2)$, i.e. two-dimensional system. Here we will find out quantity type of the isolated special points possible for a system (1) when n=3.

Let's enter linear transformation:

$$y = Bx \qquad (2)$$

where y – a new three-dimensional vector, and B – a constant square matrix of transformation. As we are interested only not in special (nondegenerate) transformation, $\det B \neq 0$, $x = B^{-1}y$, B^{-1} the return matrix to B.

Substituting (2) in (1), the system (1) can give a look:

$$\frac{dy}{dt} = Cy + yf^k(B^{-1}y), \; (3)$$

where $C = BAB^{-1}$. (4)

The system (3) has the same appearance, as a system (1), but a matrix of coefficients changed on formula (4).

It is natural to try at the set matrix A to pick up a matrix B so that matrix with took, whenever possible, simpler form. In usual courses it is proved that the matrix can always give type of a Jordan normal form.

When all elementary dividers of the first degree (so will be if all roots of the characteristic equation $\det(A - \lambda E) = 0$ various), matrix with will have a diagonal appearance where E – a single matrix.

Let the system (1) be reduced to a look (2) and a matrix with -diagonal. If to pass now from a matrix notation to usual scalar record, then we will receive a canonical form of system (1)

$$\frac{dy_i}{dt} = \lambda_i y_i + y_i f^k(y_1, y_2, y_3), \quad (i = \overline{1,3}) \qquad (5)$$

where $\lambda(0) = 0$, $\lambda_j(0) \neq \lambda_k(0)$ at $j \neq k$,

$$f^k(y_1, y_2, y_3) = \sum_{l+m+n=k} a_{lmn}\, y_1^l y_2^m y_3^n.$$

Let's designate through $\lambda_i(M)$ roots of the characteristic equation which is worked out for the isolated special point $M(y_1^0, y_2^0, y_3^0)$ systems (5).

$\lambda_i(0)$ –own numbers of a matrix A, that is roots of the characteristic equation (4) which is worked out for the isolated special point of O(0, 0, 0) systems (1). a_{lmn} – constant material coefficients.

Let's consider a system (5) in cases k=2r and k=2r+1. In a case k=2r the system (5) has the following seven special points:

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O(0,0,0), $M_{1.2}\left(\pm\sqrt[2r]{\frac{\lambda_1}{a_{k00}}}\ \ 0,0\right)$, $N_{1.2}\left(0,\pm\sqrt[2r]{\frac{\lambda_1}{a_{k00}}}\ \ 0\right)$, $D_{1.2}\left(0,0,\pm\sqrt[2r]{\frac{\lambda_1}{a_{k00}}}\right)$. in the assumption that $\lambda_1 a_{k00} < 0$, $\lambda_2 a_{0k0} < 0$, $\lambda_3 a_{00k} < 0$ then it is possible to formulate the following theorem of coexistence of the isolated special points of system (3).

Theorem 1. If the condition took place k=2r, $\lambda_1 a_{k00} < 0$, $\lambda_2 a_{0k0} < 0$, $\lambda_3 a_{00k} < 0$, the system (5) has seven isolated special points, and four of them will be knots, and other three – saddles, or on the contrary, that is four of them will be saddles, and other three others – knots. In particular, if O(0,0,0) – knot, then three knots and four saddles take place; if O(0,0,0) – a saddle, then takes place four upland and three saddles.

Proof. Let's enter designations

$$P_i(y_1,y_2,y_3) = \lambda_i y_i + y_i f^k(y_1,y_2,y_3),\ \ i = \overline{1.3}. \qquad (6)$$

Then the characteristic equation for the isolated point of a system will have an appearance:

$$\begin{vmatrix} P'_{1y_1}(E)-\lambda & P'_{1y_2}(E) & P'_{1y_3}(E) \\ P'_{2y_1}(E) & P'_{2y_2}(E)-\lambda & P'_{2y_3}(E) \\ P'_{3y_1}(E) & P'_{3y_2}(E) & P'_{3y_3}(E)-\lambda \end{vmatrix} = 0. \ (7)$$

Calculating, we find that

$$\lambda_1(M_{1.2}) = -k\lambda_1(0),\quad \lambda_2(M_{1.2}) = \lambda_2-\lambda_1,\quad \lambda_3(M_{1.2}) = \lambda_3-\lambda_1, \qquad (8)$$

$$\lambda_1(N_{1.2}) = \lambda_1-\lambda_2,\quad \lambda_2(N_{1.2}) = -k\lambda_2,\quad \lambda_3(N_{1.2}) = \lambda_3-\lambda_2, \ (9)$$

$$\lambda_1(D_{1.2}) = \lambda_1-\lambda_3,\quad \lambda_2(D_{1.2}) = \lambda_2-\lambda_3,\quad \lambda_3(D_{1.2}) = -k\lambda_3. \qquad (10)$$

Let $\lambda_1(0) > \lambda_2(0) > \lambda_3(0)$, that is the beginning of coordinates of a system (5) – unstable knot. Then on formulas (8), (9), (10) we will receive inequalities:

$$\lambda_1(M_{1.2}) < 0,\quad \lambda_2(M_{1.2}) < 0,\quad \lambda_3(M_{1.2}) < 0, \ (11)$$

$$\lambda_1(N_{1.2}) > 0,\quad \lambda_2(N_{1.2}) < 0,\quad \lambda_3(N_{1.2}) < 0, \qquad (12)$$

$$\lambda_1(D_{1.2}) > 0,\quad \lambda_2(D_{1.2}) > 0,\quad \lambda_3(D_{1.2}) < 0. \ (13)$$

From inequality (11), (12), (13) we are convinced that $M_{1.2}$ – steady knot, and $N_{1.2}$, $D_{1.2}$ – saddles.

So, if O(0, 0, 0) –knot, we have three knots and four saddles in space R^3.

Let $\lambda_1(0) > \lambda_2(0) > 0, \lambda_3(0) < 0$, that is the beginning of coordinates of system (5) O (0, 0, 0) – a saddle. Then on formulas (8), (9), (10) we will receive the following inequalities

$$\lambda_1(M_{1.2}) < 0,\quad \lambda_2(M_{1.2}) < 0,\quad \lambda_3(M_{1.2}) < 0, \ (14)$$

$$\lambda_1(N_{1.2}) > 0,\quad \lambda_2(N_{1.2}) < 0,\quad \lambda_3(N_{1.2}) < 0, \qquad (15)$$

$$\lambda_1(D_{1.2}) > 0,\quad \lambda_2(D_{1.2}) > 0,\quad \lambda_3(D_{1.2}) > 0. \ (16)$$

(14), (15), (16) follows from inequality that $M_{1.2}$ – steady knot, $N_{1.2}$ – a saddle, and $D_{1.2}$ – unstable knot. In space R^3 has three saddles and four knots that was required to be proved.

Now we will consider systems (5) in a case $k = 2r + 1$, then the system (5) has the following four isolated special points:

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O(0,0,0), M $\left(\pm\sqrt[k]{-\frac{\lambda_1}{a_{k00}}}\ 0,0\right)$, N $\left(0,\pm\sqrt[k]{-\frac{\lambda_2}{a_{0k0}}}\ 0\right)$, D $\left(0,0,\pm\sqrt[k]{-\frac{\lambda_3}{a_{00k}}}\right)$ in the assumption that $a_{k00}, a_{0k0}, a_{00k} \neq 0$. Then following is fair.

Theorem 2. At $k = 2r + 1$, $a_{k00}, a_{0k0}, a_{00k} \neq 0$, a system (5) in space R^3 has only four isolated special points; from them two knots, two others – saddles.

Conclusion

The received theorems characterize a global picture of trajectories of a system (1) in Euclidean space R^3, When the system has more than one isolated special points, i.e. conditions at which in common exist a little isolated special points and types of their curves are defined.

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